Chapter 16. Storage Models

This is an introduction chapter quotation. It is offset three inches to the right.

16.1. Introduction

Types of Inventory in a Warehouse

Cycle Inventory (CI)

- Deterministic demand
- Active products

Safety Inventory (SI)

- Stochastic demand and lead-time
- Active products

Seasonal Inventory (PI)

- Active products
- Deterministic time-varying demand

Dead Inventory (DI)

- Obsolete products

16.2. Unit Load Storage Policies

Introduction

Storage Policy Definition

Storage policy is a set of rules that determines where in the warehouse arriving materials will be stored

Storage Policy Types

No Information Policies

Random (RAN), Closest-Open-Location (COL)

Product Characteristics Policies

Product Turnover Based (DED), Product Demand based (DEM), Product Inventory based (INV)

Class Product Turnover Based (CPT)

Item Characteristics Policies

Duration Of Stay (DOS)

Zone Duration of Stay (ZDOS)

Unit Load Definition

The material in this chapter will focus on unit load warehouse operations. In unit load warehouses it is assumed that all the items in the warehouse are aggregated into units of the same size and can be moved, stored, and controlled as a single entity. Typical examples of unit loads are pallets, intermodal containers, and wire baskets. It is also assumed that all the storage locations are the same size and each location can hold any unit load.

Unit Load Advantages

Uniform and Reduced Handling Operations

Uniform and Reduced Storage Operations

Reduced Information and Control

Efficient Macro Space Utilization

Unit Load Disadvantages

Cost of Assembly and Disassembly

Cost of Container

Cost of Empty Container Handling or Disposal

Inefficient Micro Space Utilization

Figure 16.1. Unit Load Pallet Rack

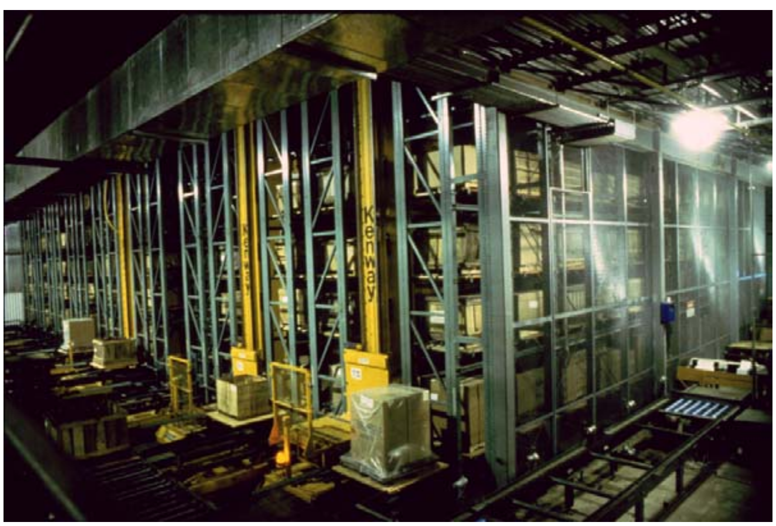


Figure 16.2. Unit Load Automated Storage/Retrieval System

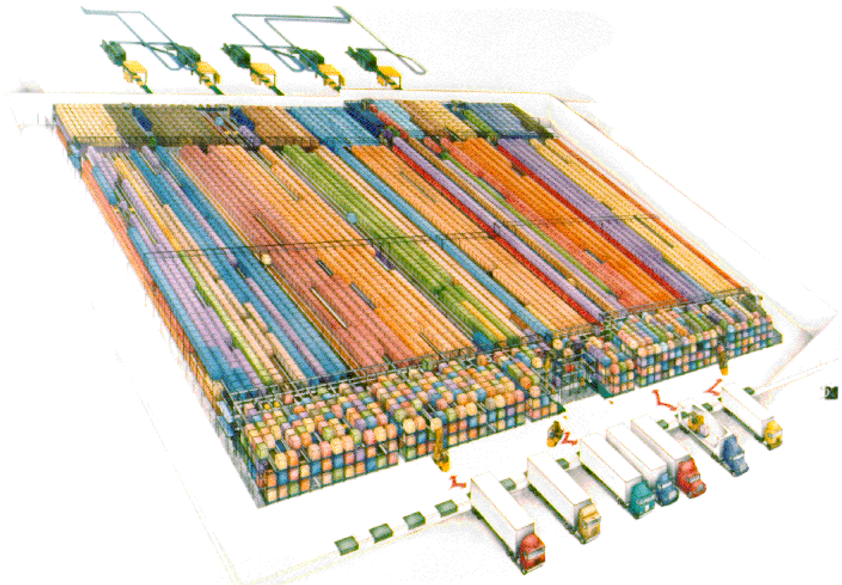


Figure 16.3. Automated Staging Warehouse for Finished Goods
(Photo courtesy of Retrotech Inc.)

Figure 16.4. Intermodal Container Carrier being Unloaded

Storage Policy Notation

I_{pt}	=	On-hand inventory of product p at time period t
N	=	Required number of locations in the warehouse when using a particular storage policy such as dedicated, shared, or maximum, which is indicated by the subscript

q_p	=	Replenishment quantity of product p, also called the cycle inventory of product p
s_p	=	Safety inventory quantity of product p
r_p	=	Demand rate for product p
r_{pk}	=	Demand rate for product p entering or leaving through warehouse dock k
p_{pk}	=	Probability for a unit of product p to enter or leave through warehouse dock k
f_p	=	Frequency of access of a location in zone p or assigned to product p
e_j	=	Expect one-way travel time to location j
t_{jk}	=	Travel time to location j from warehouse dock k
e_{pj}	=	Expected one-way travel time for a unit load of product p stored in location j
T_p	=	Total travel time for product p or zone p during the planning horizon

By definition, the following relationships exist

$$r_p = \sum_k r_{pk} \text{ and } p_{pk} = \frac{r_{pk}}{r_p} \tag{16.1}$$

$$e_{pj} = \sum_k p_{pk} \cdot t_{jk} \tag{16.2}$$

Command Cycle

Command Cycle is the Number of Operations Performed on a Single Trip

•Single Command

•Dual Command

•Multiple Command (Order Picking)

Cycle Time is the Expected Time to Complete a Single Cycle.

It is assumed that all the operations in the warehouse are performed in single command mode, i.e. the picker or crane performs a single operation on each round trip.

Travel Independence or Factoring Condition

If the travel independence or factoring condition is satisfied, then it is assumed that all the items in the warehouse have the same probability mass function for selection of a dock or input/output point. This allows the computation of the expected one-way distance for each location, independent of which unit load will be stored in that location. The travel independence condition is equivalent to

$$p_{pk} = p_k \ \forall p \ \text{ or } \ e_j = e_{pj} \ \forall p \tag{16.3}$$

The expected one-way distance for each location can then be computed as

$$e_j = \sum_k p_k \cdot t_{jk} \tag{16.4}$$

The travel independence condition was first specified by Malette and Francis (1972) under the name *factoring condition*, since, if this condition is satisfied, the expected travel time to a particular location holding a particular product can be factored or computed as the product of the expected travel time of the location and the frequency of access of the product.

Warehouse Operations Objectives

Minimize the Expected Travel Time

Minimize MH Equipment and Personnel

$$Min \ \sum_j f_j \cdot t_j \tag{16.5}$$

Minimize the Required Storage Space

Minimize Capital Investment

$$Min \ \ N \tag{16.6}$$

Maximize Flexibility

Minimize Material Handling and Operations by Humans

Usually Conflicting Objectives

Main Warehousing Facilities Design Principle (Travel Time)

Place unit loads that generate the highest frequency of access in locations with the lowest expected access time.

Figure 16.5. Frequency of Access Curve for Warehouse Operations

Main Warehousing Facilities Design Principle (Storage Capacity)

Use the "Cube" by utilizing the height of the warehouse and keeping it filled.

This principle encourages the use of the vertical dimension of the warehouse and the avoidance of empty unit locations. The vertical dimension of the warehouse can be used with block stacking storage systems and a large variety of rack storage systems. Empty unit storage locations can be avoided by the proper storage policy.

Shared versus Dedicated Storage Policies

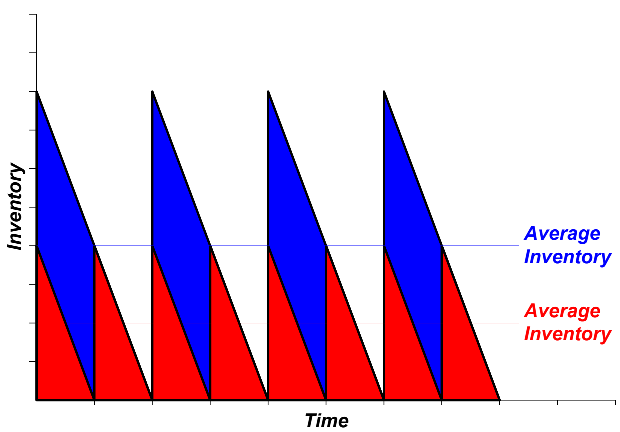


Figure 16.6. Cyclic Inventory Pattern

Dedicated Storage

With dedicated storage, a set of locations is reserved for the items of a single product during the entire planning period. The required warehouse size N is equal to the sum of the maximum inventories of each product. The location and management of items can be done by hand under relative stable demand conditions.

Product Dedicated Storage Policies Characteristics

Static

Simple

Space inefficient (maximum)

Unconstrained replenishment

$$N_{DED} = \sum_{p} \left(q_p + s_p \right) = \sum_{p} \max_{t} \left\{ I_{pt} \right\} = N_{MAX} \tag{16.7}$$

Shared Storage

With shared storage, a location can be used successively for the storage or items of different products. Examples of shared storage are random and closest open location storage. The required warehouse size N is equal to the maximum over time of the aggregate inventory. For many uncorrelated products, this size is half the size required by dedicated storage. Shared storage requires almost always a computerized system to manage and locate items in the warehouse, but this system has larger flexibility in adapting to changing demand conditions. Throughput comparisons depend on which shared storage policy is used, i.e. dedicated storage does not always minimize the expected travel time. This last statement is contrary to what is taught in many courses and is still controversial.

Product Shared Storage Policies Characteristics

Dynamic

Requires inventory map

Simple (COL) or complex (DOS)

Space efficient

$$N_{SHA} = \max_{t} \left\{ \sum_{p} I_{pt} \right\} \tag{16.8}$$

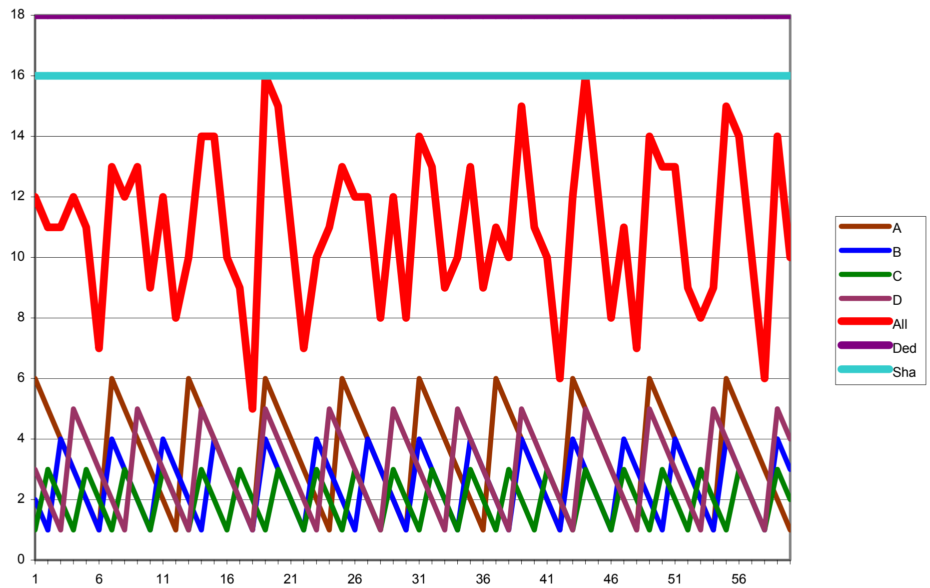


Figure 16.7. Warehouse Size for Unbalanced Product Flows under Various Storage Policies

The ratio of the required warehouse size to the maximum required size under dedicated storage is called the sharing factor α. The sharing factor has a range of [0.5, 1]. This sharing factor can be most easily determined by simulation.

$$\alpha = \frac{N}{N_{DED}} \in [0.5, 1] \tag{16.9}$$

Similarly, a warehouse balance β can be computed as

$$\beta = 2(1-\alpha)$$
$$\alpha = 1 - \frac{\beta}{2} \tag{16.10}$$

The sharing factor and warehouse balance indicate how well balanced the input and output flows of the warehouse are. A value of $\alpha = 1$ or $\beta = 0$ indicates that the flows are not balanced at all. A value of $\alpha = 0.5$ or $\beta = 1$ indicates that the flows are perfectly balanced.

Consider the following example of a perfectly balanced warehouse holding four products, each with a replenishment quantity of four unit loads and no safety inventory. The required storage size under a shared storage policy such as random or closest-open-location is 10 locations. The required storage size under product-dedicated storage is 16 locations. This yields the following balance characteristics for this (unrealistic) warehouse. The on-hand inventory for the four products and all products combined is shown in Figure 16.8.

$$\alpha = \frac{10}{16} = 0.6250$$
$$\beta = 2\left(1 - 0.6250\right) = 0.75$$

Figure 16.8. Warehouse Size for Balanced Product Flows under Various Storage Policies

Product Based Dedicated Storage

Consider the following warehouse layout. The warehouse has four rows of bays, with six bays in each row for a total of 24 bays. All the bays are 10 by 10 feet and only one product is stored per bay.

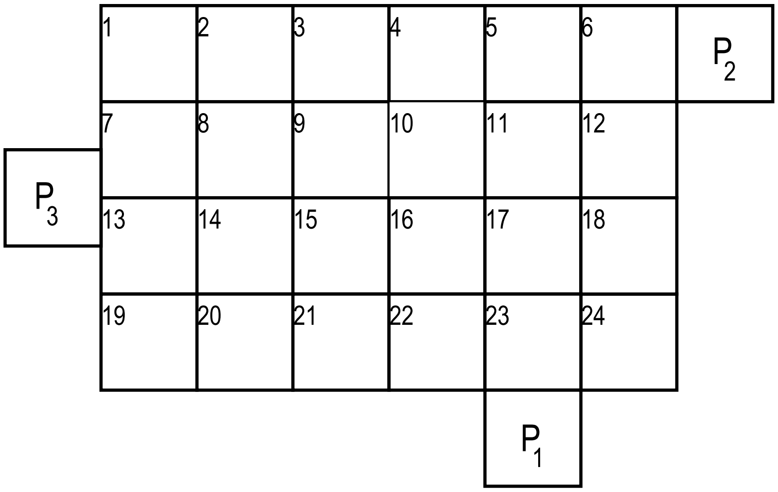


Figure 16.9. Warehouse Layout

Case 1: Factoring

All the material is received through the receiving door labeled P_3. Material is shipped through the two shipping doors labeled P_1 and P_2. All receipts and shipments are full pallet quantities. The following product information has been provided.

Table 16.1. Product Information (Factoring Case)

product	A	B	C
product storage requirements (q)	12	2	10
pallets received per month (r)	400	60	200
pallets shipped per month through			
door P1 (p1)	300	45	150
door P2 (p2)	100	15	50

The distance computations use a rectilinear distance norm between the centroid of each storage bay and the centroid of shipping/receiving areas. The warehouse operates under single command. A month is assumed to be 30 days.

We will first show the expected one-way travel distance for each location in the warehouse for the combined storage and retrieval of a single unit load. Then we will determine the best assignment of products to storage bays. Finally, we will compute the total travel per month for each product and for the total warehouse system.

Case 1 corresponds to a dedicated storage policy with the travel independence or factoring condition satisfied. It can be solved by hand by the innerproduct minimization of the frequency of access and expected travel time vectors by sorting them in opposite directions.

The probability mass functions for the three products are all equal to (0.375, 0.125, 0.500), hence the travel independence conditions is satisfied.

The expected travel one-way travel distance for location j is then given by:

$$e_j = \sum_{k=1}^{K} p_k t_{kj} \tag{16.11}$$

For example, $e_1 = 0.375 \cdot 80 + 0.125 \cdot 60 + 0.500 \cdot 25 = 50$ and $e_{13} = 0.375 \cdot 60 + 0.125 \cdot 80 + 0.500 \cdot 15 = 40$.

If a single unit square is moved the delta is 3.75, 1.25, and 5.00, with respect to the first, second, and third dock, respectively. Moving from square 1 to square 2 to gives a delta of -3.75-1.25+5.00=0. Moving from square 5 to square 6 gives a delta of 3.75-1.25+5.00=7.5. Moving from square 1 to square 7 gives a delta of -3.75+1.25-0.50=-7.5. Moving from square 7 to square 13 gives a delta of -3.75+1.25=-2.5. Finally, moving from square 7 to square 19 gives a delta of -3.75+1.25+5.00=2.5. The resulting expected travel distances are shown at the bottom of the unit squares in Figure 16.10.

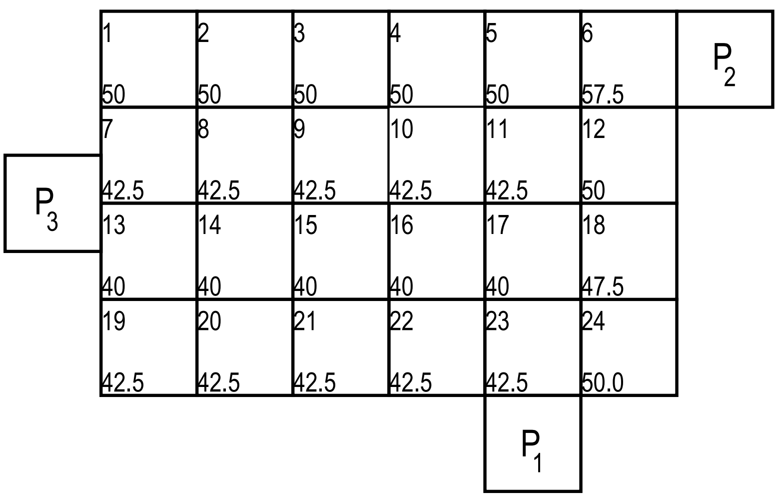


Figure 16.10. Expected one-way travel distances for all products (Factoring Case)

There are three dedicated product-based storage policies that are used very frequently in practice. They are often described by the following catch phrases.

1. Fast and furious or "Fast movers closest to the door"
2. Small is beautiful or "Small inventory closest to the door"
3. But higher turns beats them all, where the frequency of access is equal to the ratio of demand rate divided by maximum inventory, or "Fastest turning closest to the door"*Product Turnover-Based*

To find the order in which to locate products, the "frequency-of-access" of each product must be computed as the ratio of monthly demand divided by number of bays, i.e.

$$f_A = \frac{r_A}{q_A} \tag{16.12}$$

This is similar to the "cube-per-order-index" introduced by Heskett (1963, 1964). This policy was proven to be optimal for product-based dedicated storage by Malette and Francis (1972). Finally, Malmborg and Bhaskaran (1987) showed that it is also the optimal policy if the warehouse operates under a mixture of single and dual command cycles.

The frequencies of access for the products are:

$f_A = 400/12 = 33.33$

$f_B = 60/2 = 30$

$f_C = 200/10 = 20$

The products are assigned by decreasing frequency of access to the locations by increasing expected travel time. This is equivalent to minimizing the innerproduct of two vectors by sorting them in opposite directions. Hence product *A* get assigned first, then product *B*, and finally product *C*. The assignments are shown in the Figure 16.11.

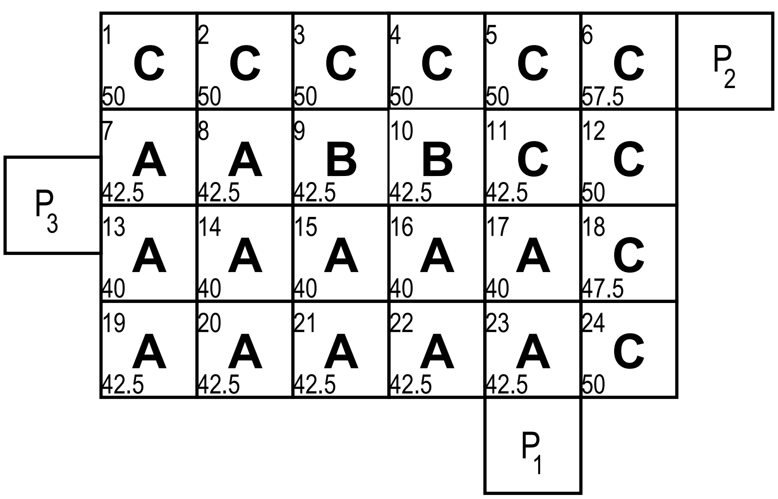


Figure 16.11. Assignment Solution for the Factoring Case

The total travel cost per month (time period) is computed first by product and then summed over all products. Let Z_A be the set of locations associated with product A and let t_A be the average one-way travel distance to a location assigned to product *A*, then the total travel for product *A* under single command cycles is given by:

$$T_A = 4 r_A t_A = 4 r_A \left(\frac{\sum_{j \in \mathbf{Z}_A} e_j}{q_A} \right) = 4 f_A \left(\sum_{j \in \mathbf{Z}_A} e_j \right) \tag{16.13}$$

$$T = \sum_{p=1}^{P} T_p \tag{16.14}$$

The one-way distance is multiplied by a factor 4 to compute the time for the two round-trip material handling moves, one for storage of the unit load and one for retrieval of the unit load.

The total travel distances per month for the products are then:

$T_A = 4 \cdot 400 \cdot (5 \cdot 40 + 7 \cdot 42.5)/12 = 4 \cdot 400 \cdot 497.5/12 = 4 \cdot 400 \cdot 41.46 = 66{,}333$
$T_B = 4 \cdot 60 \cdot (2 \cdot 42.5)/2 = 4 \cdot 60 \cdot 42.5 = 10{,}200$
$T_C = 4 \cdot 200 \cdot (42.5 + 47.5 + 7 \cdot 50 + 57.5)/10 = 4 \cdot 200 \cdot 497.5/10 = 4 \cdot 200 \cdot 49.75 = 39{,}800$
$T = T_A + T_B + T_C = 66{,}333 + 10{,}200 + 39{,}800 = 116{,}333$

Two commonly used storage policies are based on sequencing the products by decreasing operations or by increasing required storage space. This first policy is commonly denoted by "putting the fast movers closest to the door." The second policy could be referred to as "putting the low inventory products closest to the door." The average monthly travel time for each policy will be computed for the above example.

Demand Based

The "fast movers" are identified by the largest demand or, equivalently, by the largest number of operations. The products are then sorted by decreasing number of operations. The number of operations for the three products are:

$f_A = 400, \;\; f_C = 200, \;\; f_B = 60$

The products are assigned by decreasing number of operations to the locations by increasing expected travel time. This is equivalent to minimizing the innerproduct of two vectors by sorting them in opposite directions. Hence product *A* get assigned first, then product *C*, and finally product *B*. The assignments are shown in the Figure 16.12.

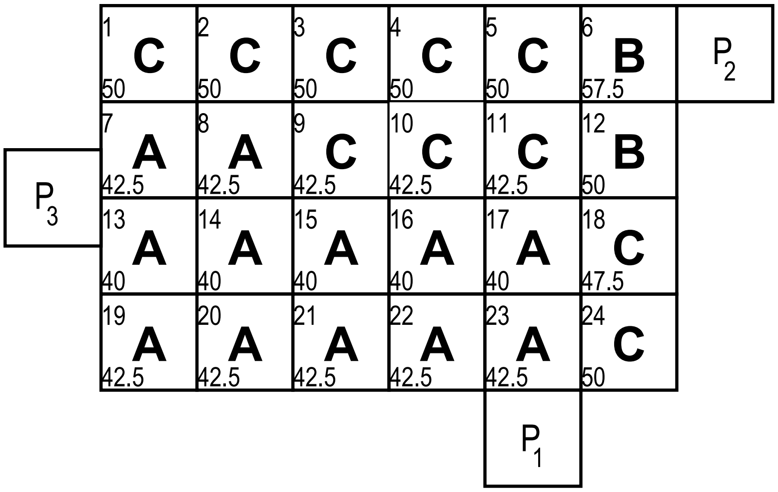


Figure 16.12. Warehouse Layout for the Factoring Case based on Demand

The total travel cost per month (time period) is computed first by product and then summed over all products.

The total travel distances per month for the products are then:

TA = 4·400·(5·40+7·42.5)/12 = 4·400·497.5/12 = 4·400·41.46 = 66,333

TB = 4·60·(50+57.5)/2 = 4·60·107.5/2= 4·60·53.75 = 12,900

TC = 4·200·(3·42.5+47.5+6·50)/10 = 4·200·475.0/10 = 4·200·47.50 = 38,000

T = TA + TB + TC = 66,333 + 12,900 + 38,000 = 117,233

This is an increase of 0.8 % over the optimal turnover-based dedicated storage policy.

Inventory Based

The "low inventory" or "small inventory" products are identified by their required number of storage locations. The products are then sorted by increasing number of storage locations. The number of storage locations for the three products are:

$f_A = 12,\; f_C = 10,\; f_B = 2$

The products are assigned by decreasing number of operations to the locations by increasing expected travel time. This is equivalent to minimizing the innerproduct of two vectors by sorting them in opposite directions. Hence product *B* get assigned first, then product *C*, and finally product *A*. The assignments are shown in the Figure 16.13.

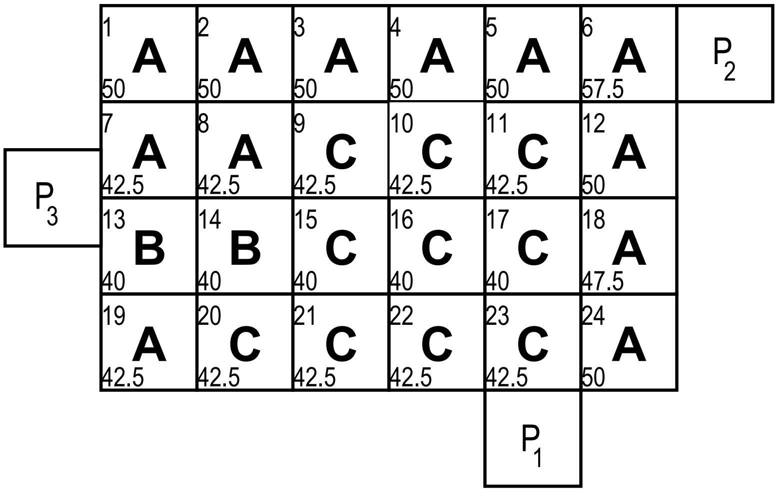


Figure 16.13. Warehouse Layout for the Factoring Case based on Inventory

The total travel cost per month (time period) is computed first by product and then summed over all products.

The total travel distances per month for the products are then:

$TA = 4 \cdot 400 \cdot (3 \cdot 42.5 + 47.5 + 7 \cdot 50 + 57.5)/12 = 4 \cdot 400 \cdot 582.5/12 = 4 \cdot 400 \cdot 48.54 = 77{,}667$

$TB = 4 \cdot 60 \cdot (2 \cdot 40)/2 = 4 \cdot 60 \cdot 80/2 = 4 \cdot 60 \cdot 40 = 9{,}600$

$TC = 4 \cdot 200 \cdot (3 \cdot 40 + 7 \cdot 42.5)/10 = 4 \cdot 200 \cdot 417.5/10 = 4 \cdot 200 \cdot 41.75 = 33{,}400$

$T = TA + TB + TC = 77{,}667 + 9{,}600 + 33{,}400 = 120{,}667$

This is an increase of 3.7 % % over the optimal turnover-based dedicated storage policy.

Case 2: Non-Factoring

Consider the following shipping pattern, with the same storage requirements as given before.

Table 16.2. Product Information (Non-Factoring Case)

Product	A	B	C
Product storage requirements (q)	12	2	10
Pallets received and shipped per month through			
door P_1	300	6	100
door P_3	100	24	240
door P_2	400	90	60

Case 2 corresponds to a dedicated storage policy where the travel independence or factoring condition is not satisfied. It must be solved with a transportation or assignment model.

The probability mass functions for the three products are given in the following table:

Table 16.3. Probability mass functions in percent (Case 2).

product	A	B	C
door P1 (p_1)	37.5	5	25
door P2 (p_2)	12.5	20	60
door P3 (p_3)	50	75	15

The probability mass function for product A is the same as in case 1, and hence the expected one way travel distances for product A are given in Figure 16.2. The expected one way travel distances for products B and C are computed in a similar manner and are given in Figures 16.14 and 16.15, respectively.

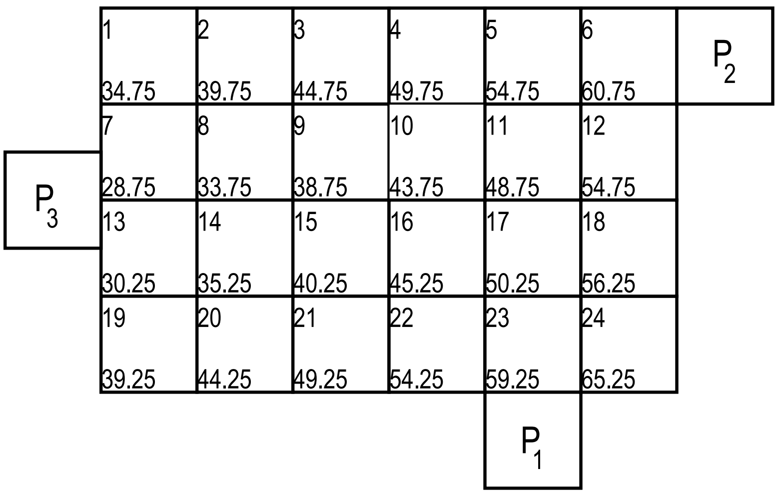


Figure 16.14. Expected One-way Travel Distances for Product B

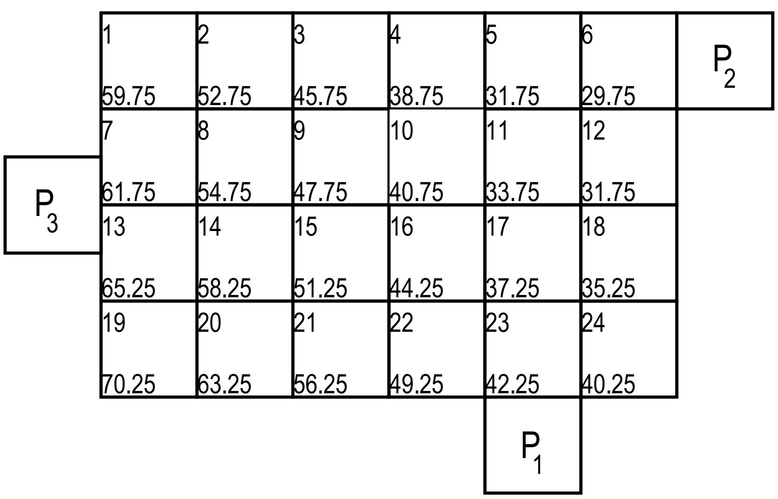


Figure 16.15. Expected One-way Travel Distances for Product C

The linear transportation formulation is then:

$$
\begin{aligned}
Min \quad & \sum_{i=1}^{M}\sum_{j=1}^{N} f_i e_{ij} x_{ij} \\
s.t. \quad & \sum_{j=1}^{N} x_{ij} = q_i \\
& \sum_{i=1}^{M} x_{ij} \le 1 \\
& x_{ij} \ge 0
\end{aligned}
\tag{16.15}
$$

A linear programming model, compatible with the Take command of LINDO or the Read LP format of CPLEX, is given next. The results of the linear programming model are shown in Figure 16.16.

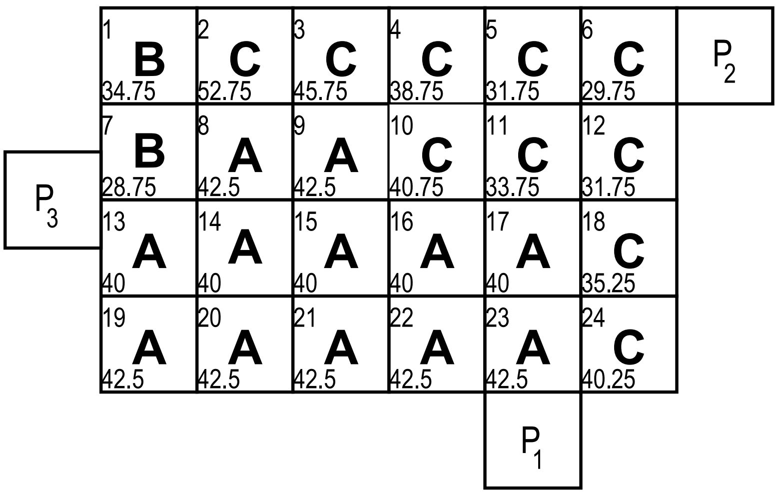


Figure 16.16. Assignment Solution for the Non-Factoring Case

The total travel times for the products are then:

$T_A = 4 \cdot 400 \cdot 497.5/12 = 66{,}333.33$

$T_B = 4 \cdot 60 \cdot 63.5/2 = 7620$

$T_C = 4 \cdot 200 \cdot 380.50/10 = 4 \cdot 200 \cdot 38.05 = 30{,}440$

$T = T_A + T_B + T_C = 66{,}333.33 + 7{,}620 + 30{,}440 = 104{,}393.33$

The factoring storage policy for the same three products with the same total demand but with different material handling moves requires 11.44 % more travel time than the non-factoring policy.

Linear Programming Model for Non-Factoring Case

Let x_{ij} be equal to one if the product *i* is assigned to location *j*. Let *SGI* be the sum of the one-way travel distances to all locations assigned to product *I*. The objective is to minimize innerproduct of the frequency of access of each product multiplied by the total travel distance for all locations assigned to that product. The first three constraints are the definition of the *SGI*'s for each product. The next three constraints ensure that there are enough locations assigned to each product. Finally, the last constraints ensure that each location holds at most one unit load.

Code Listing 1. Non-Factoring Storage Policy LP Formulation

```
MIN 133.3333 SGA + 120 SGB + 80 SGC
SUBJECT TO
50 XA1 + 50 XA2 + 50 XA3 + 50 XA4 + 50 XA5 + 57.5 XA6 +
42.5 XA7 + 42.5 XA8 + 42.5 XA9 + 42.5 XA10 + 42.5 XA11 + 50 XA12 +
40 XA13 + 40 XA14 + 40 XA15 + 40 XA16 + 40 XA17 + 47.5 XA18 +
42.5 XA19 + 42.5 XA20 + 42.5 XA21 + 42.5 XA22 + 42.5 XA23 + 50 XA24
- SGA = 0
34.75 XB1 + 39.75 XB2 + 44.74 XB3 + 49.75 XB4 + 54.75 XB5 + 60.75 XB6 +
28.75 XB7 + 33.75 XB8 + 38.75 XB9 + 43.75 XB10 + 48.75 XB11 +
54.75 XB12 + 30.25 XB13 + 35.25 XB14 + 40.25 XB15 + 45.25 XB16 +
50.25 XB17 + 56.25 XB18 + 39.25 XB19 + 44.25 XB20 + 49.25 XB21 +
54.25 XB22 + 59.25 XB23 + 65.25 XB24
- SGB = 0
59.75 XC1 + 52.75 XC2 + 45.75 XC3 + 38.75 XC4 + 31.75 XC5 + 29.75 XC6 +
61.75 XC7 + 54.75 XC8 + 47.75 XC9 + 40.75 XC10 + 33.75 XC11 +
31.75 XC12 + 65.25 XC13 + 58.25 XC14 + 51.25 XC15 + 44.25 XC16 +
37.25 XC17 + 35.25 XC18 + 70.25 XC19 + 63.25 XC20 + 56.25 XC21 +
49.25 XC22 + 42.25 XC23 + 40.25 XC24
- SGC = 0
XA1 + XA2 + XA3 + XA4 + XA5 + XA6 + XA7 + XA8 + XA9 + XA10 +
XA11 + XA13 + XA14 + XA15 + XA16 + XA17 + XA18 + XA19 + XA20 + XA21 +
XA22 + XA23 + XA24 + XA12 = 12
XB1 + XB2 + XB3 + XB4 + XB5 + XB6 + XB7 + XB8 + XB9 + XB10 +
XB11 + XB13 + XB14 + XB15 + XB16 + XB17 + XB18 + XB19 + XB20 + XB21 +
XB22 + XB23 + XB24 + XB12 = 2
XC1 + XC2 + XC3 + XC4 + XC5 + XC6 + XC7 + XC8 + XC9 + XC10 +
XC11 + XC13 + XC14 + XC15 + XC16 + XC17 + XC18 + XC19 + XC20 + XC21 +
XC22 + XC23 + XC24 + XC12 = 10
XA1 + XB1 + XC1 <= 1
XA2 + XB2 + XC2 <= 1
XA3 + XB3 + XC3 <= 1
XA4 + XB4 + XC4 <= 1
XA5 + XB5 + XC5 <= 1
XA6 + XB6 + XC6 <= 1
XA7 + XB7 + XC7 <= 1
XA8 + XB8 + XC8 <= 1
XA9 + XB9 + XC9 <= 1
XA10 + XB10 + XC10 <= 1
XA11 + XB11 + XC11 <= 1
XA12 + XB12 + XC12 <= 1
XA13 + XB13 + XC13 <= 1
XA14 + XB14 + XC14 <= 1
XA15 + XB15 + XC15 <= 1
XA16 + XB16 + XC16 <= 1
XA17 + XB17 + XC17 <= 1
XA18 + XB18 + XC18 <= 1
XA19 + XB19 + XC19 <= 1
XA20 + XB20 + XC20 <= 1
XA21 + XB21 + XC21 <= 1
```

```
XA22 + XB22 + XC22 <= 1
XA23 + XB23 + XC23 <= 1
XA24 + XB24 + XC24 <= 1
END
```

Product Turnover Class Based Storage

Pure Dedicated is Very Space Inefficient

3 to 5 Classes based on Frequency of Access

Dedicated Space for Each Class

Inside Class Use Random or Closest Open Location

Shared Storage Policy

Class Space Determined by Simulation or approximated based on service levels and statistics.

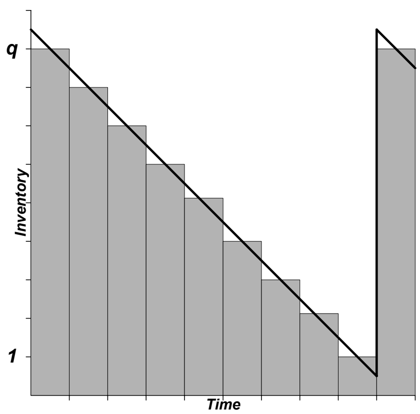


Figure 16.17. Inventory Distribution for a Constant Demand Rate

Note that the expected value and variance of a uniformly distributed random variable between the boundary values *a* and *b* is equal to

$$\begin{aligned} \bar{x} &= \frac{b+a}{2} \\ \sigma^2 &= \frac{(b-a)^2}{12} \end{aligned} \qquad (16.16)$$

Assuming there is no safety inventory present, the mean and variance of the inventory of product *p* is then

$$\bar{I}_p = \frac{q_p + 1}{2}$$
$$\sigma_p = \frac{(q_p - 1)}{\sqrt{12}} \tag{16.17}$$

Based on the central limit theorem, if the class ***K*** contains a reasonable large number of products, the total inventory in class ***K*** is normally distributed with mean and variance equal to

$$\bar{I}_K = \sum_{p \in K} \bar{I}_p$$
$$\sigma_K = \sqrt{\sum_{p \in K} \sigma_p^2} \tag{16.18}$$

The required zone size dedicated to a particular class can then be determined given the acceptable probability that the zone will be full when a unit load of the class arrives. Let α be the maximum acceptable probability that a zone will be full, then

$$P\left[\frac{x - \bar{x}}{\sigma} \geq z\right] \leq \alpha$$
$$Z_K = \bar{I}_K + z \cdot \sigma_K \tag{16.19}$$

The safety inventory s must be added to cycle inventory q to compute the locations required for product p. The safety and cycle inventory are determined by supply chain factors such as the cost tradeoff between inventory and transportation and the required level of customer service.

$$f_p = \frac{r_p}{q_p + s_q} \tag{16.20}$$

$$\bar{I}_p = \frac{q_p + s_p + 1}{2}$$
$$\sigma_p = \frac{(q_p - 1)}{\sqrt{12}} \tag{16.21}$$

Duration-of-Stay Shared Storage

Illustration

General Parameters

4 Products (A, B, C, D)

Replenishment Batch Size q = 4 Unit Loads

Demand Rate r = 1 Unit Load / Day

Replenishment Days (A = 1, B = 2, C = 3, D = 4)

AS/RS Storage with simultaneous travel in both directions ($v_x = v_y = 1$), so the Chebyshev travel time is used.

$$t^{CHEB} = \max\left\{\frac{\Delta_x}{v_x}, \frac{\Delta_y}{v_y}\right\} = L_\infty$$

	3	3	3	4
	2	2	3	4
	1	2	3	4
I/O	1	2	3	4

Figure 16.18. Storage Example Rack Travel Times

Product-Turnover Based Dedicated Storage

Each Product Turnover Rate = 1/4

Any Storage Assignment is Optimal

Maximum Storage Space = 16

Average Daily Travel = (10 + 10 + 11 + 13) / 4 = 11

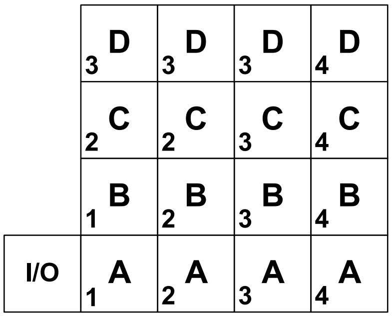


Figure 1619. Product Dedicated Optimal Storage

Unit-Load Duration-of-Stay Storage

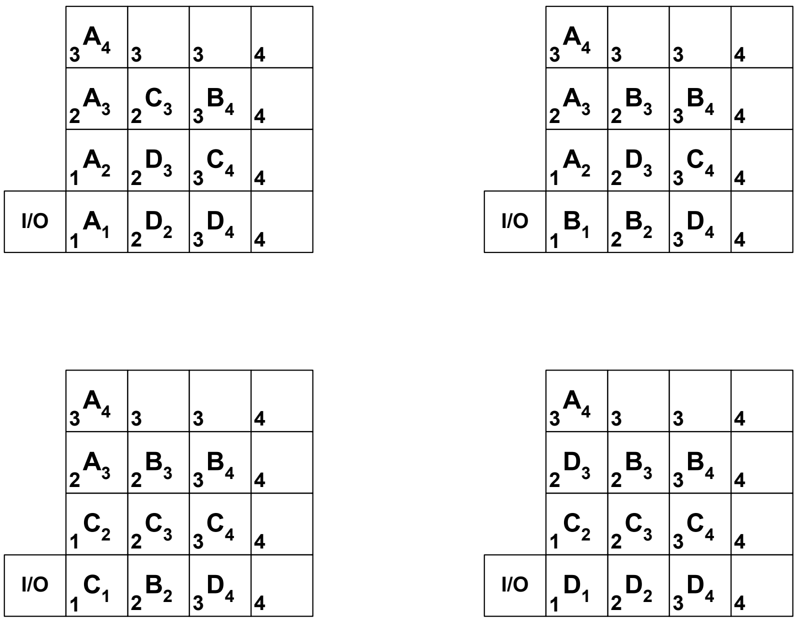


Figure 1620. Duration-Of-Stay Storage Patterns On the Different Days

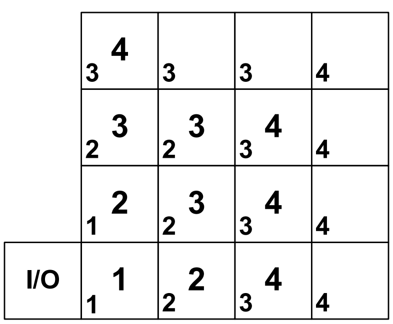


Figure 16.21. Duration-Of-Stay Optimal Storage

Store by Increasing "Duration-Of-Stay" (DOS)

Required Storage Space = 10 is minimum

Average Daily Travel = (1 + 3/2 + 6/3 + 12/4) = 7.5

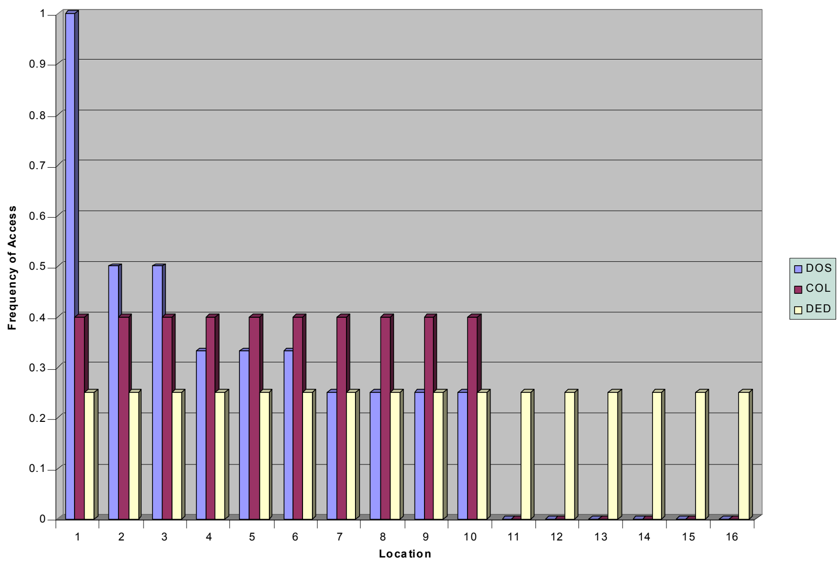


Figure 16.22. Frequency of Access Distribution for Various Storage Policies

Required Storage Space: 10 Shared, 16 Dedicated, + 60 %

Expected Travel Time: 7.5 Shared, 11 Dedicated, + 47 %

Accesses to Best Location: 1 Shared, 0.25 Dedicated, -75 %

Exploits that First and Last Unit Load in Batch are Different

Cross Docking (DOS = 0)

Minimizes Both Storage Space and Travel Time for a Perfectly Balanced Warehouse

Very Constrained Perfectly Balanced Replenishment Pattern $n_p(t)$

Perfectly Balanced Warehouse

$n_{DOS}(t)$ = Number of unit loads arriving during time period t that have a duration of stay equal to *DOS*

z_{DOS} = Size of the zone, expressed as a number of storage locations, reserved for the storage of unit loads with duration of stay equal to *DOS*

Balanced = Minimum Space

$$n_{in}(t) = n_{out}(t) \qquad \forall t \tag{16.22}$$

Perfectly Balanced = Minimum Space and Minimum Time

$$n_{DOS}(t) = n_{DOS}(t + DOS) \qquad \forall t, \forall DOS$$
$$z_{DOS} = \sum_{i=1}^{DOS} n_{DOS}(i) \qquad (16.23)$$

A perfectly balanced warehouse implies that the warehouse is also balanced.

$$n_{DOS,in}(t) = n_{DOS,in}(DOS + t) \qquad \forall t, \forall DOS$$
$$n_{DOS,out}(t + DOS) = n_{DOS,in}(DOS + t) \qquad \forall t, \forall DOS$$
$$n_{DOS,out}(t) = n_{DOS,in}(t) \qquad \forall t, \forall DOS$$
$$\sum_p n_{DOS,out}(t) = \sum_p n_{DOS,in}(t) \qquad \forall t$$
$$n_{out}(t) = n_{in}(t) \qquad \forall t$$

Example

Consider the following product data and the same warehouse layout. All the material is received through the receiving door labeled P_3. Material is shipped through the two shipping doors labeled P_1 and P_2. For each product three times as much material is shipped through door P_1 as through door P_2. All receipts and shipments are full pallet quantities. The warehouse operates under single command. The travel time is measured centroid to centroid with the rectilinear distance norm. All the bays are 10 by 10 feet and only one item is stored per bay. The best shared storage policy is used. Products are replenished during the day their inventory reaches zero. The expected one-way travel distance for each location in the warehouse for the combined storage and retrieval of a single unit load is computed as shown in Figure 16.1023

Table 16.4. Product Information (Example 2).

product	daily demand	reorder quantity	replenishment day
A	1	4	3
B	0.25	2	2
C	1	4	2
D	1	4	1
E	0.25	3	3
F	0.25	3	7
G	1	4	4
H	0.25	2	6
I	0.25	3	11

There are three groups of products that have the same daily demand and reorder quantity. The groups consist of products A, C, D, and G (group 1), products B and H (group 2), and products E, F, and I (group 3). First we check if each of the groups satisfies the perfectly balanced condition. For instance, for group 2 on day 2 a unit load with duration of stay 4 and 8 is withdrawn and product B is replenished which deposits a unit load with duration of stay 4 and 8 days. So group 2 is perfectly balanced. Similar computations show that each group does satisfy the perfectly balance condition. Next we construct the input/output diagram by duration of stay shown in Table 16.5.

Table 16.5. Input/Output Diagram by Duration of Stay

	1	2	3	4	5	6	7	8	9	10	11	12	zone
1	D	C	A	G									1
2	D	C	A	G									2
3	D	C	A	G									3
4	D	C, B	A, E	G		H	F				I		6
5													0
6													0
7													0
8		B	E			H	F				I		4
9													0
10													0
11													0
12			E				F				I		3

The last column shows how large the zones have to be for each of the durations of stay. Only the unit loads in the first p days have to be summed to find the zone size for loads of duration of stay p, since at period p+1 the pattern repeats itself. The relevant unit loads are shown in bold in Table 16.5 to the left of and below the staircase line. The resulting warehouse layout is shown in Figure 24. The number in each bay indicates the duration of stay of any load stored in this bay, no longer the product label. The travel times are then computed first by duration of stay zone and then for the whole warehouse. The total number of slots used is equal to 19, even though the total number of slots required for dedicated storage would have been 29. Notice that not all the storage bays are used when using the shared storage policy. In fact, for the perfectly balanced case, the number of storage bays used is the smallest possible. In addition, the average travel is also minimized.

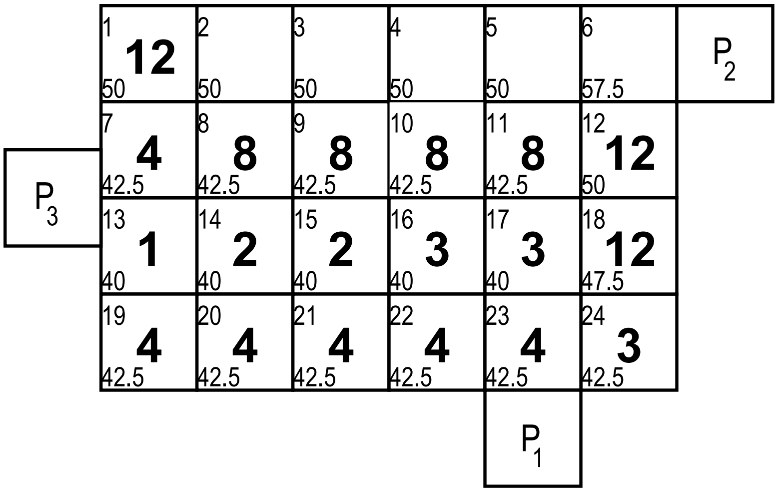


Figure 16.24 Duration of Stay Warehouse Zones

The travel distance per duration of stay (DOS) zone is then computed with the following formula, where z_{DOS} is the size of the duration of stay zone Z_{DOS} and t_{DOS} is the average one-way travel distance to a location in this zone.

$$T_{DOS} = 4\frac{1}{DOS} z_{DOS} t_{DOS} = \frac{4}{DOS} z_{DOS} \left(\frac{\sum_{j \in \mathbf{Z}_{DOS}} e_j}{z_{DOS}} \right) = \frac{4}{DOS} \left(\sum_{j \in \mathbf{Z}_{DOS}} e_j \right) \tag{16.24}$$

$$T = \sum_{DOS} T_{DOS} \tag{16.25}$$

$$T_1 = 4 \cdot 40 / 1 = 160$$
$$T_2 = 4 \cdot (40 + 40) / 2 = 160$$
$$T_3 = 4 \cdot (40 + 40 + 42.5) / 3 = 163.33$$
$$T_4 = 4 \cdot (6 \cdot 42.5) / 4 = 255$$
$$T_8 = 4 \cdot (4 \cdot 42.5) / 8 = 85$$
$$T_{12} = 4 \cdot (47.5 + 2 \cdot 50) / 12 = 49.17$$
$$T = T_1 + T_2 + T_3 + T_4 + T_8 + T_{12} = 872.50$$

Further information can be found in Goetschalckx and Ratliff (1990).

Not Perfectly Balanced Warehousing Systems

Static Greedy Heuristic

Sort by Increasing Departure Time

Adaptive, Dynamic Heuristic

Combine DOS into classes

Remedial Action for Full Classes

$$z_{DOS} = DOS \cdot E\left[n_{DOS}\right] \tag{16.26}$$

Safety Stock

Assuming, that the unit loads of a product follow the first-in-first-out (FIFO) withdrawal rule, any safety inventory present increases the amount of time unit loads of an arriving batch will spend in the warehouse.

For example, assume that the replenishment batch size is 36 pallets and that the average demand or withdrawal rate is 4 pallets a day. If there is no safety inventory present, when the current batch arrives, then the duration of stay of the pallets in the batch will form the following series

$$\frac{1}{4}, \frac{2}{4}, \frac{3}{4}, \ldots \frac{36}{4}$$

If there are eight pallets of safety inventory in the warehouse when the new batch arrives, then the duration of stay of the pallets in this batch will form the following series

$$\frac{8+1}{4}, \frac{8+2}{4}, \frac{8+3}{4}, \ldots \frac{8+36}{4}$$

In general, the duration of stays of the unit loads without and with safety stock are given by the following series, respectively.

$$DOS_1 = \frac{1}{r}, DOS_2 = \frac{2}{r}, \ldots DOS_q = \frac{q}{r} \tag{16.27}$$

$$DOS_1 = \frac{s+1}{r}, DOS_2 = \frac{s+2}{r}, \ldots DOS_q = \frac{s+q}{r} \tag{16.28}$$

Closest-Open-Location and Random Shared Storage

For comparison purposes we can also compute the expected travel distance under closest open location storage policy. The closest open location storage policy is equivalent to the pure random storage policy if all locations in the rack are used. For our example, the required number of locations is the same as under perfectly balanced shared storage, since the input and output flows are identical. The 19 locations with lowest travel distance will be used and the average travel distance will be based only on those 19 locations.

$$T_i = 4 f_i q_i \left(\frac{\sum_{j=1}^{N} e_j}{N} \right) = 4 f_i q_i t_i = 4 r_i t_i \qquad (16.$$

$$T_{RAN} = \sum_i T_i = 4 t_i \sum_i r_i \qquad (16.$$

d = (5 · 40 + 10 · 42.5 + 47.5 + 3 · 50) / 19 = 822.50 / 19 = 43.29

T = 4 · 43.29 · (4 · 1 + 5 · 0.25) = 909.08

Comparison of Storage Policies

Comparison Example

Given a warehouse configuration as shown in the next Figure with a total of 18 locations and three docks. Each location and each dock is assumed to be 10 feet wide by 10 feet long. The travel is assumed to be rectilinear from location centroid to location centroid and the warehouse is assumed to operate under single command. It is assumed that the loads can travel through the dock areas if required. The product data are given in the following Table. All products depart through dock P3, 20 % of all products arrive through dock P1, 20 % of all products arrive through dock P2, and 60 % of all products arrive through dock P3.

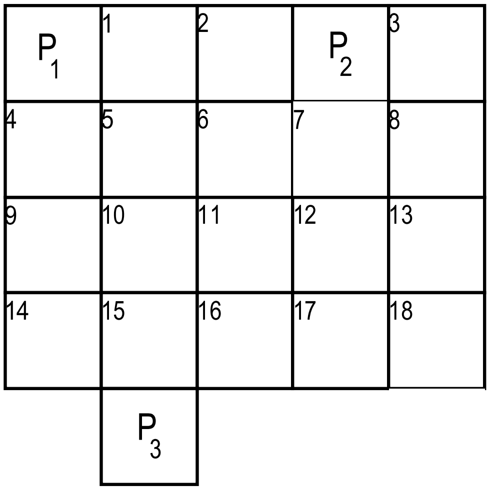


Figure 16.25. Warehouse Layout for Comparison of Storage Policies

Table 16.6. Comparison of Storage Policies Product Data

Product	Demand Rate	Reorder Quantity	Re supply Period
A	0.5	2	2
B	1	2	2
C	1	3	3
D	1	2	1
E	0.5	2	4
F	1	3	2
G	1	3	1

The solution procedure will execute a sequence of computations to arrive at the optimal warehouse layout under the various storage policies and single command and will finally compare the performance of the policies.

First, the dock selection probability mass functions are computed for each product and the travel independence condition is verified. Since all products have the same interface pattern with the docks the travel independence condition is satisfied and there is only one probability mass function. Its values are:

$p_1 = (0.2 + 0) / 2 = 0.1$

$p_2 = (0.2 + 0) / 2 = 0.1$

$p_3 = (0.6 + 1.0) / 2 = 0.8$

Second, the expected round trip storage and retrieval travel distances for each location are computed.

$e_1 = 4 \cdot (0.1 \cdot 10 + 0.1 \cdot 20 + 0.8 \cdot 40) = 4 \cdot 35 = 140$

$e_2 = 4 \cdot (0.1 \cdot 20 + 0.1 \cdot 10 + 0.8 \cdot 50) = 4 \cdot 43 = 172$

$e_3 = 4 \cdot (0.1 \cdot 40 + 0.1 \cdot 10 + 0.8 \cdot 70) = 4 \cdot 61 = 244$

$e_4 = 4 \cdot (0.1 \cdot 10 + 0.1 \cdot 40 + 0.8 \cdot 40) = 4 \cdot 37 = 148$

$e_7 = 4 \cdot (0.1 \cdot 40 + 0.1 \cdot 10 + 0.8 \cdot 50) = 4 \cdot 45 = 180$

The difference in total distance when moving one location “down” is $4 \cdot (1 + 1 - 8) = 4 \cdot (-6) = -24$. This allows the easy computation of the total distances for the rest of the locations. The results are shown in the next Figure.

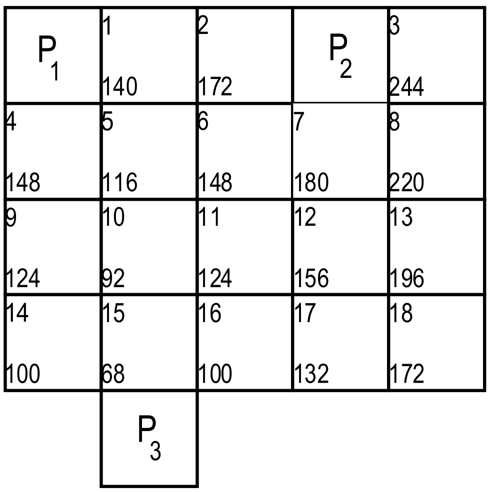


Figure 16.26. Expected Storage and Retrieval Round Trip Travel Distances

Third, the frequency of access for each product under product turnover dedicated storage is computed and the products are ranked by non-increasing frequency of access. Then the locations are assigned to the products based upon this ranking. If there are ties in the selection of locations, then products are kept together as much as possible. Breaking these ties will lead to alternative warehouse layouts that have the same overall distance score.

Table 16.7. Frequency of Access Computation and Rank

Product	Demand Rate	Reorder Quantity	Re supply Period	Frequency of Access	Rank
A	0.5	2	2	0.25	3
B	1	2	2	0.5	1
C	1	3	3	0.33	2
D	1	2	1	0.5	1
E	0.5	2	4	0.25	3
F	1	3	2	0.33	2
G	1	3	1	0.33	2

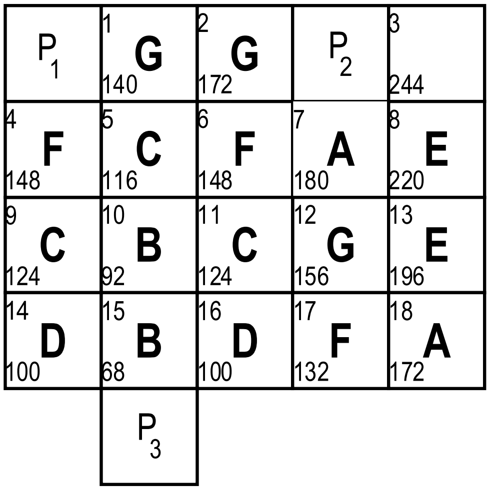


Figure 16.27. Product Storage Layout

Fourth, the total travel distance per products is computed and then these total travel distances are added to yield the overall travel for the product turnover dedicated warehouse layout. Observe also that the required warehouse size for dedicated storage is equal to the sum of the reorder quantities, which is 17 in this example.

$T_A = 0.25 \cdot (172 + 180) = 88$

$T_B = 0.5 \cdot (68 + 92) = 80$

$T_C = 0.33 \cdot (124 + 124 + 116) = 121.33$

$T_D = 0.5 \cdot (100 + 100) = 100$

$T_E = 0.25 \cdot (196 + 220) = 104$

$T_F = 0.33 \cdot (132 + 148 + 148) = 142.67$

$T_G = 0.33 \cdot (140 + 156 + 172) = 156$

$T = 88 + 80 + 121.33 + 100 + 104 + 142.67 + 156 = 792$

Fifth, we verify that each group of products is perfectly balanced. Then we construct the table showing the number of unit loads of each product with their arrival period and duration of stay. Summing the first p days for each duration of stay p then yields the required zone size (i.e. number of locations) for that duration of stay. The results are given in the following table. The total required warehouse size is equal to the sum of the zone size, which is equal to 12 in this example.

Table 16.8. Input/Output Diagram by Duration of Stay

	1	2	3	4	zone
1	D,G	B,F	C		2
2	D,G	A,B,F	C	E	5
3	G	F	C		3
4		A		E	2

Sixth, the optimal layout for duration of stay storage policies is determined by assigning the zones with the smallest duration of stay to the locations with the lowest expected travel distances. The results are shown in the next Figure.

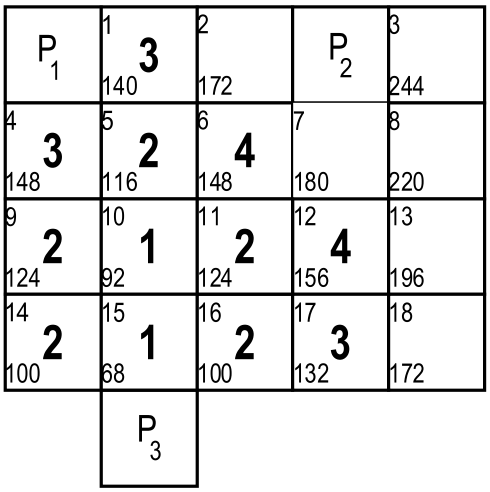


Figure 16.28. Unit Duration-Of-Stay Warehouse Layout

Seventh, the total travel distance for the duration of stay storage policy is computed by first computing the total travel distance per duration of stay zone and than adding all these travel distances together.

$T_1 = (1/1)\cdot(68 + 92) = 160$

$T_2 = (1/2)\cdot(100 + 100 + 124 + 124 + 116) = 282$

$T_3 = (1/3)\cdot(148 + 140 + 132) = 140$

$T_4 = (1/4)\cdot(148 + 156) = 76$

$T = 160 + 282 + 140 + 76 = 658$

Eight and last, the space and travel distance ratios for the product-turnover dedicated storage and the duration-of-stay shared storage are computed.

$$\frac{T_{DOS}}{T_{DED}} = \frac{658}{792} = 83\%$$

$$\frac{N_{DOS}}{N_{DED}} = \frac{12}{17} = 70\%$$

Comparison of Storage Policies Experiment

Policies: DOS, COL, DED, 2CL, 2 ZN

Products: 10, 20, 40, 80

Batch Size: 5, 10, 20, 40

3 Replications

Random First Replenishment Period

Simulation of Deterministic System

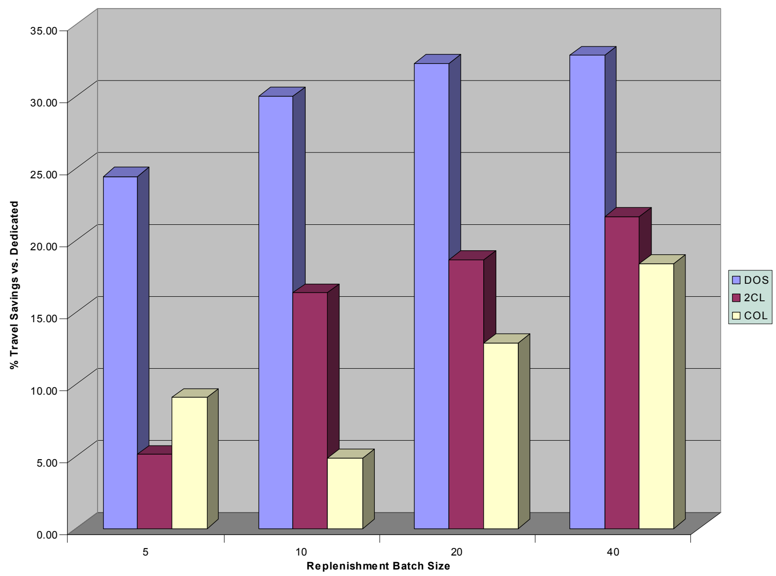


Figure 16.29. Influence of the Batch Size on the Performance of Storage Policies

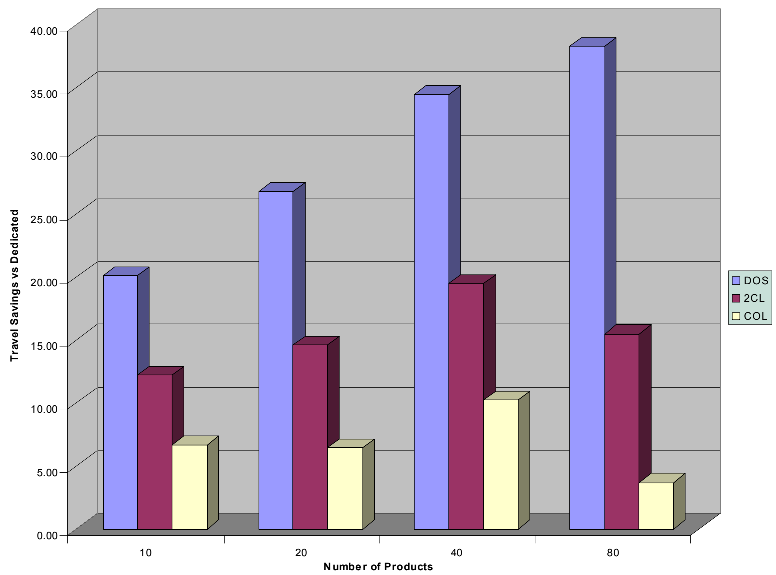


Figure 16.30. Influence of the Number of Products on the Performance of Storage Policies

Experimental Comparison Summary

Adaptive DOS is Superior (20 - 30 % Savings)

Two Class is Next Best (15 % Savings)

Two Zone is not as good as expected

Full Turnover Dedicated is Worst of All

Storage Policy Conclusions

Real Systems are Not Perfectly Balanced

Duration of Stay Reduces Travel and Storage Space

2 Class Product Performs Well

Savings Magnitude Depends on Replenishment Pattern

Data Requirements Indicate Automated Warehouses

16.3. Pick versus Reserve Storage Policies

Introduction

Many warehouses are divided into two distinct functional zones. In the first zone, the most frequently demanded items are stored in a storage system where items can be accessed in a high-speed manner. This zone is called the pick zone. Because the storage system is usually expensive, the number of items that can be stored in the pick zone is limited. The second zone holds large quantities of items that are not as frequently accessed. This zone is called the reserve zone. The storage capacity of the reserve zone is for most practical purposes unlimited. Some items can be stored in both the pick and the reserve zone. They are picked from the pick zone and, when necessary, restocked from the reserve zone through an internal replenishment. Typical material handling and storage systems for the pick zone are a flow rack, automated A or V frame order picking systems, and bin shelving. Typical material handling and storage systems for the reserve zone are pallet rack and case shelving.

Whenever an item is retrieved from the picking zone rather than from the reserve zone savings are realized. Because the pick zone is small and items stored in it are easily accessible, the cost for picking a single item from it is less than for picking an item from the reserve zone. On other hand, items stored in the pick zone require the extra handling step associated with the internal replenishment. These two costs must be traded off for each product while taking in consideration the total storage capacity of the pick zone. The warehouse manager must then decide if and how much of each product to store in the pick zone.

To make that decision, it is assumed that each product has a dedicated storage space in the pick zone. Products in the pick zone can then be replenished independently of each other, i.e., the pick zone operates under a dedicated storage policy. It is also assumed that the savings in pick costs and the cost of a single internal replenishment of each product is independent of the quantity of the products stored in the pick zone.

The storage capacity of the pick zone and the amount of product stored must be expressed in the same units and represent the critical storage resource of the pick zone. For bin shelving or a gravity flow rack, the critical resource is the area each product takes up of the face of the rack. The storage capacity of the pick zone is then the total rack face area. For an automated A or V frame order picking system, the critical resource is the length along the conveyor belt that a product takes up. The storage capacity of the pick zone is then twice the total length of the pick frame.

Formulation

The following notation will be used:

$y_i =$ binary decision variable indicating if product *i* is stored in the pick zone or not

$x_i =$ continuous decision variable indicating the amount of critical resource space is allocated to item *i*

$X =$ critical storage capacity of the pick zone

$N =$ total number of products in the warehouse

$D_i =$ number of requests per unit time for item *i*

$R_i =$ the demand per unit time for item *i* expressed in critical storage units

$c_i =$ cost per internal replenishment of item *i*

$e_i =$ savings per request for item *i* if the item is stored in the pick zone

The decisions of which items and how much of each item to store in the pick zone can then be determined by solving the following formulation.

$$\begin{aligned} Max. \quad & \sum_{i=1}^{N} y_i \cdot \left(e_i D_i - \frac{c_i R_i}{x_i} \right) \\ s.t. \quad & \sum_{i=1}^{N} x_i \le X \\ & y_i \in \{0,1\} \\ & x_i \ge 0 \end{aligned} \qquad (16.$$

Observe that the objective function is concave for all positive values of x_i but that the optimal x_i may be zero if it is not profitable to assign item *i* to the pick zone. The above formulation is a typical knapsack problem, which is known to be NP-complete. Hence, it is very unlikely that an efficient optimal solution algorithm can be found for the very large problem instances that typically occur in the pick-versus-reserve problem.

Heuristic

The above problem was studied by Hackman and Rosenblatt (1990). They proposed the following heuristic procedure.

Assume that we know which items are to be stored in the pick zone. In other words, the optimal values of the y variables have already been determined. We then need to determine how much of each item to store, subject to the overall capacity constraint. This is the space allocation subproblem.

Let $I^+ = \{i | y_i = 1\}$ be the given collection of items that are to be stored in the pick zone, then the optimal quantities to be stored can be determined with the following formulation

$$\begin{aligned} Max. \quad & z = \sum_{i \in I^+} \frac{c_i R_i}{x_i} \\ s.t. \quad & \sum_{i \in I^+} x_i \le X \qquad [\lambda] \\ & x_i > 0 \end{aligned} \qquad (16.$$

The optimal solution to this formulation must satisfy the Kuhn-Tucker conditions, or

$$\lambda^* = \frac{\partial z}{\partial x_i} = \frac{c_i R_i}{\left(x_i^*\right)^2}$$

or

$$x_i^* = \sqrt{\frac{c_i R_i}{\lambda^*}}$$

Since the objective function is increasing in function of x_i, the capacity constraint will be satisfied as an equality, or

$$\sum_{i \in I^+} x_i^* = X$$

which yields

$$\lambda^* = \left(\frac{1}{X} \sum_{i \in I^+} \sqrt{c_i R_i} \right)^2 \qquad (16.$$

and

$$x_i^* = \frac{\sqrt{c_i R_i}}{\sum_{i \in I^+} \sqrt{c_i R_i}} X \qquad (16.$$

However, the items that are to be located in the pick zone by the optimal solution are not known. Since the original problem is known to be NP-complete and thus difficult to solve to optimality for large problem instances, a heuristic solution will be used. The standard heuristic for solving this problem is to rank the items by the highest "bang-for-the-buck" ratio, i.e., by decreasing ratio of

$$\frac{e_i D_i - \frac{c_i R_i}{x_i}}{x_i} = \frac{e_i D_i}{\sqrt{c_i R_i}} \sqrt{\lambda^*} - \lambda^*$$

The sequence of the items will remain the same if we rank the items based on the non-increasing ratio

$$\frac{e_i D_i}{\sqrt{c_i R_i}}$$

If the savings from picking from the active pick area and the cost for the internal replenishment to the active pick area are the same for all products, then this ratio can be further simplied to the following ratio.

$$\frac{D_i}{\sqrt{R_i}}$$

In other words, products should be ranked by decreasing ratio of their number of picks divided by the square root of the volume flow over the planning horizon.

Algorithm 16.1. Hackman-Rosenblatt Pick versus Reserve Heuristic

1. Sort items by non-increasing $\frac{e_i D_i}{\sqrt{c_i R_i}}$ ratio. Break ties by placing items with highest denominator first.
2. For each ordered set of items $S_k = \{1,2,...k\},\ 1 \le k \le N$,
   compute the optimal space allocation with $x_i^* = \frac{\sqrt{c_i R_i}}{\sum_{i \in I^+} \sqrt{c_i R_i}} X$ and compute the objective function value
   $z = \sum_{i \in I^+} e_i D_i - \frac{c_i R_i}{x_i}$
3. Keep the set of items S_k with maximum value of z. Break ties by selecting the set with the smallest cardinality.

Normally, one would have to solve N subproblems to find the best set S_k. However, Hackman and Rosenblatt showed that the function $z(S_k)$ is unimodal with respect to k. Hence, a linear search, such as the bisection or Golden Section, reduces the number of subproblems that need to be solved to $O(\log_2 N)$.

Example

In most storage systems the savings per pick and the cost of replenishment are independent of the product being picked or replenished. In addition, the critical resource is usually the volume capacity of the storage system. The following notation will be used:

v_i = fraction of the total storage system volume capacity allocated to product *i*

V = storage system volume capacity

f_i = volume flow of product *i* during the planning horizon, expressed in units such as cubic feet per year

The storage system capacity is then allocated according to the square root of flow

$$v_i^* = \frac{\sqrt{f_i}}{\sum_{i \in I^+} \sqrt{f_i}} V \qquad (16.$$

A product with twice the volume flow will get 41 % more space in the storage system. Compare the following two products

Product	A	B
Yearly Demand	5200 units/year	260 units/year
Items per Pick	100	1
Picks per Year	52	260
Volume per Item	4 in^3	64 in^3
Volume Flow per Year	$5200*4/12^3=12.04$ ft^3	$260*64/12^3=9.63$ ft^3
Space Fraction	$\frac{\sqrt{12.04}}{\sqrt{12.04}+\sqrt{9.63}} = 0.53$	$\frac{\sqrt{9.63}}{\sqrt{9.63}+\sqrt{12.04}} = 0.47$

Storage Mode Allocation Procedure

This procedure was developed by Bartholdi and Hackman.

Traditionally, the storage mode has been determined based on simple rules that took in consideration the cubic volume and the number of picks of a product. The following figure was presented in Frazelle (1997).

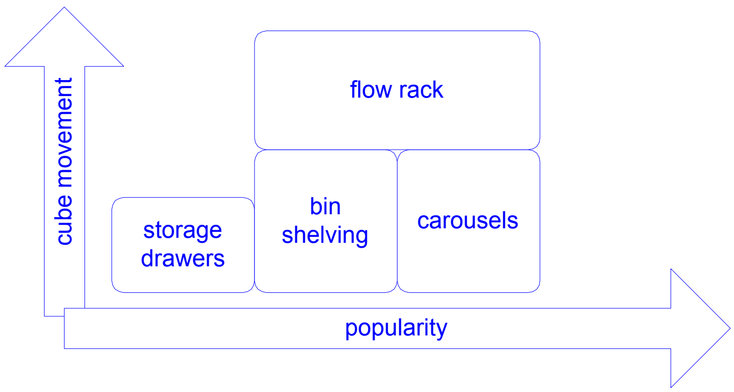


Figure 16.31. Rule-Based Storage Mode Assignment

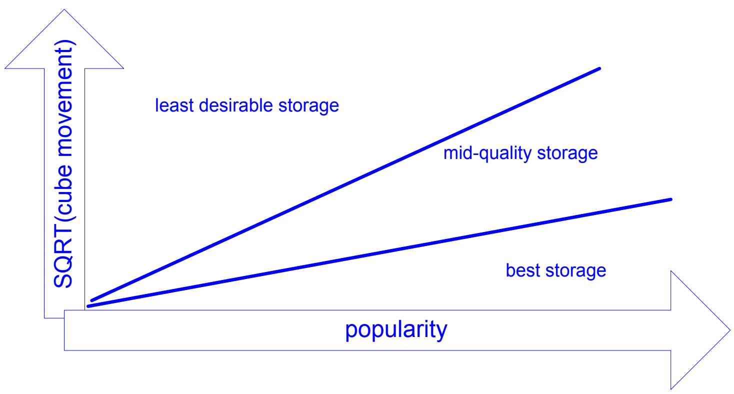


Figure 16.32. Rule-Based Storage Mode Assignment

16.4. Block Stacking Storage systems

Introduction

Figure 16.33. Block Stacking Storage Example

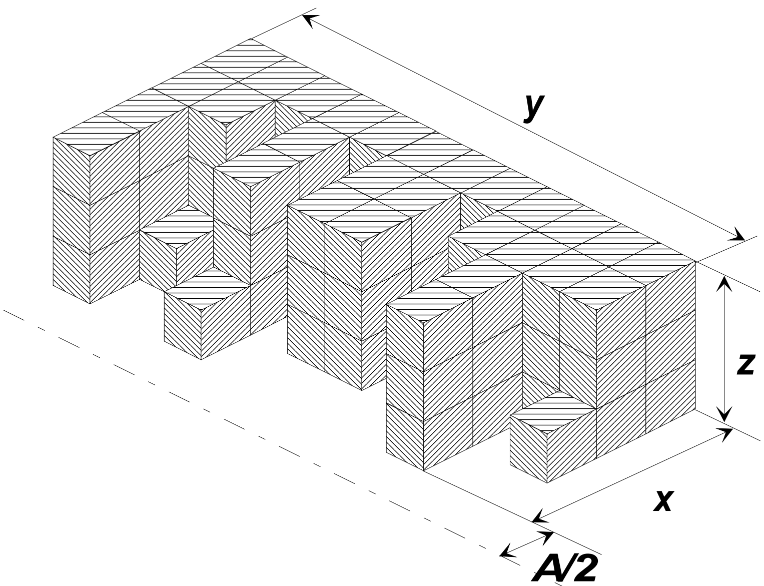


Figure 16.16.34 Block Stacking Illustration

Block Stacking Applications

Few Products

Large Quantities

Palletized or Boxed Products

Without Supporting Rack Structures

Figure 16.35. Block Stacking Storage Example

Figure 16.36. Block Stacking Storage Example

Figure 16.37. Block Stacking Storage Example

Block Stack Characteristics

Stackable Products

High Storage Density

Limited Investment

Limited Product Variety

Block Stacking Terminology

Unit Load

Stack

Lane

Aisle

Single Command Unit Load Storage and Retrieval

Block Stacking Apllication Areas

Finished Goods Warehouse

Distribution Warehouse

Public Warehousing

Block Stacking Objectives

1. Maximize Space Utilization
2. Maximize Storage Flexibility
3. Minimize Transportation Costs

To Determine Optimal or Near-Optimal Lane Depths for Single and Multiple Products That Maximize the Space Utilization in Block Stacking Storage Systems And Minimize Honeycomb Space Loss

Basic Space-Time Tradeoff

Travel Aisle Space + Storage Lane Space versus Time this Space is Occupied

Required Decision Policies

Storage Policy for Arriving Loads in a Product Batch

Warehouse Layout Design

Assumptions

LIFO by Lane

FIFO by Product

No Mixed Lanes

No Relocations

Constant Demand Rate

Instantaneous replenishment

Perfectly Balanced Shared Storage

Perfectly Balanced Shared Storage = Whenever a Lane of Depth is Vacated,
a Product Requiring a Lane of that Depth has Arrived

Matson and White (1981,1984) studied extensively the case of a single lane depth for all products in the warehousing system. Goetschalckx and Ratliff (1991) derived a computation procedure for the optimal multiple lane depths in the warehouse and compared this with various heuristic lane depths.

The following notation is used:

Q	=	number of unit loads in batch
W	=	pallet width along the aisle
L	=	pallet length perpendicular to the aisle
A	=	travel aisle width
I	=	safety stock in pallets at time of arrival
d	=	constant demand rate
z	=	stack height in unit loads
x	=	lane depth vector
y	=	number of lanes per depth
N	=	total number of lanes
r_n	=	number of stacks in lanes n+1 through N

The parameters and decision variables are illustrated in the following Figure of a block stacking ground plan.

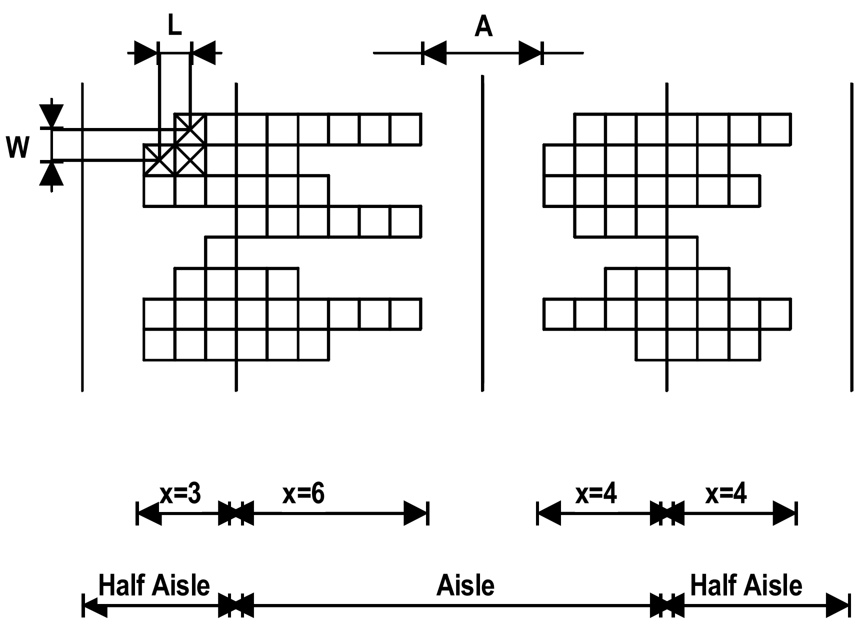


Figure 16.38 Block Stacking Ground Plan

Single Lane Depth Systems

Basic Space-Time Tradeoff

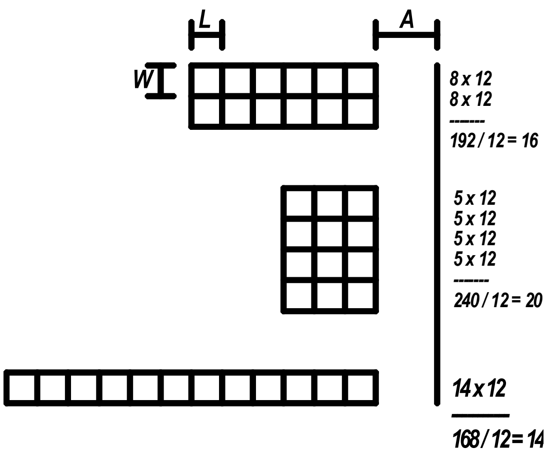


Figure 16.39 Dedicated Storage Time-Space Tradeoff

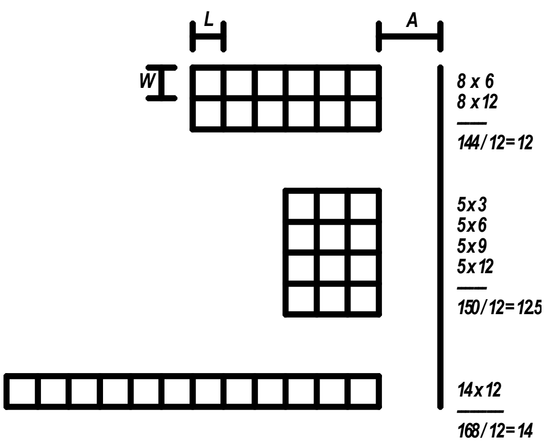


Figure 16.40 Shared Storage Time-Space Tradeoff

Single Lane Depth Derivation for a Single Product

Literature Review

Kind (1965, 1975)

$$x = \sqrt{\frac{QA}{Lz}} - \frac{A}{2L} \tag{16.38}$$

Matson and White (1981, 1984)

$$x = \sqrt{\frac{(Q+2I)A}{2Lz}} \tag{16.39}$$

Optimal Single Lane Depth

Since the number of lanes has to be an integer number, it is computed using the ceiling function, which in essence rounds up the number of lanes if required:

$$y = \left\lceil \frac{Q}{xz} \right\rceil \tag{16.40}$$

The total space-time requirement can be computed by multiplying the square area of each lane with the time this lane is occupied. The footprint area of each lane and its associated aisle space is equal to

$$W(xL + 0.5A) \tag{16.41}$$

The occupation time of the first incomplete lane is:

$$t_1 = \frac{I + [Q - (y-1)zx]}{d} \tag{16.42}$$

The occupation time for the second through yth lane is then:

$$\begin{aligned} t_2 &= t_1 + \frac{xz}{d} \\ t_3 &= t_1 + \frac{2xz}{d} \\ t_y &= t_1 + \frac{(y-1)xz}{d} \end{aligned} \tag{16.43}$$

The sum of the occupation times is then:

$$\begin{aligned} \sum_{j=1}^{y} t_j &= yt_1 + \frac{xz}{d}\sum_{j=1}^{y-1} j \\ &= \frac{y\left[(Q+I) - (y-1)zx\right]}{d} + \frac{y(y-1)xz}{2d} \end{aligned} \tag{16.44}$$

The total space-time requirement is then given by

$$S = \frac{y(xL + 0.5A)W[2(Q+I) - (y-1)xz]}{2d} \tag{16.45}$$

S is a non-convex function of the lane depth x because x and y both must have integer values. If we consider the continuous relaxation of the problem where x and y no longer have to be integer and the product of xyz is exactly equal to Q, then S_c becomes a convex function of x.

$$S_c = \frac{QW(xL + 0.5A)(Q + 2I + xz)}{2dxz} \tag{16.46}$$

Computing the first derivative and setting it equal to zero yields the optimal continuous single lane depth x^*_c. The second derivative is also computed and always larger than zero for non-zero lane depths, which proves that S_c is a convex function of the lane depth x.

$$\frac{dS_c}{dx} = \frac{QW}{d}\left(\frac{L}{2} - \frac{A(Q+2I)}{4x^2 z}\right)$$
$$\frac{d^2 S_c}{dx^2} = \frac{QWA(Q+2I)}{2dzx^3} > 0 \qquad (16.47)$$

$$x_c^* = \sqrt{\frac{(Q+2I)A}{2Lz}} \qquad (16.48)$$

Since S is non-convex for the original problem, the optimal continuous single lane depth is not necessarily the optimal single lane depth. To find the optimal lane depth all possible lane depths are evaluated with complete enumeration. This can be easily done with a spreadsheet. This will be illustrated in the next section for multiple products.

Single Lane Depth Derivation for a Multiple Products

The determination of the optimal single lane depth for multiple products can be best captured in the following table. For each product and for each lane depth x the required number of lanes for the product is computed with Formula 16.40 and the total space-time requirement is computed with Formula 16.45.

The example considers a warehouse with two products. For both products the length and the width of a pallet including all clearances are equal to 4 feet and the travel aisle is 16 feet wide. For the first product A, the number of pallets in the replenishment batch is equal to 60, the stack height is equal to 3 pallets. The daily demand rate is equal to 0.5 pallets/day. There is no initial safety stock for this product. For the second product B, the number of pallets in the replenishment batch is equal to 60, the stack height is equal to 5 pallets. The daily demand rate is equal to 0.25 pallets/day. There is no initial safety stock for this product. The lane depth computations are shown in the next table. Observe that the best single lane depth for product A is 5 stacks and the best single lane depth for product B is either 4 or 6 stacks, but that the best single lane depth for both products together is 6 stacks.

Table 16.9. Single Lane Depth for Multiple Products

x	y_A	S_A	y_B	S_B	SS_p
1	20	60480	12	74880	135360
2	10	42240	6	53760	96000
3	7	36960	4	48000	84960
4	5	34560	3	46080	80640
5	4	33600	3	47040	80640
6	4	33792	2	46080	79872
7	3	33696	2	48960	82656
8	3	34560	2	51200	85760
9	3	34848	2	52800	87648
10	2	34560	2	53760	88320
11	2	36192	2	54080	90272
12	2	37632	1	53760	91392

The non-convex nature of the space-time curve is illustrated in the following figure, which corresponds to the values in the above table. The space-time value for product B and lane depth 5 is higher than for both lane depths 4 and 6.

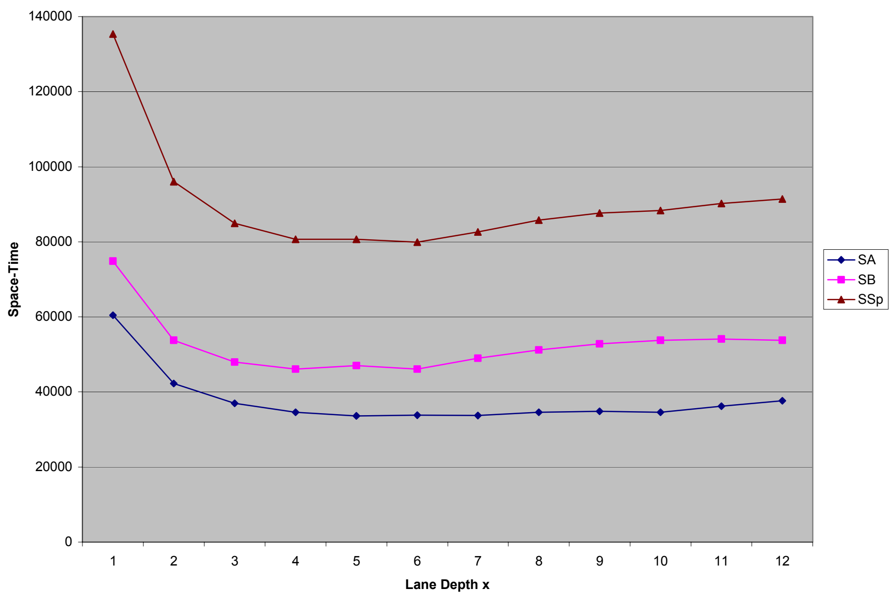


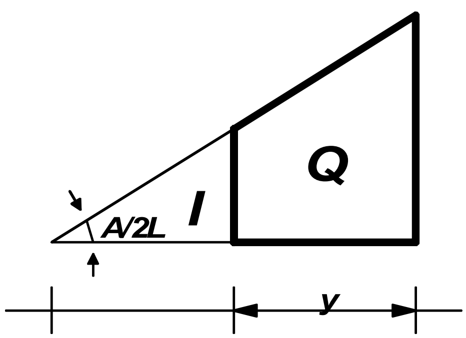
Figure 41. Space-Time Curves for Multiple Products

Multiple Lane Depths Systems

Goetschalckx and Ratliff (1991) developed a method based on dynamic programming to derive the optimal multiple lane depths for a product.

If the lane depth and the number of lanes are modeled by continuous variables, then the optimal lane depths for a product increase linearly with a growth rate of $A/2L$. The lane depths growth pattern is illustrated in the following figure.

If there is safety stock remaining in the warehouse when the next batch of the product arrives, then the optimal lanes depths still have the same linear growth rate but the first lane depth for the new batch is determined as if the newly arriving inventory is appended to lane depths for the safety stock. It should be noted that the safety stock is not stored in shallow lane depths at the tip of the triangle, just that the lane depths for the new batch behaves like if it were. So, the more safety inventory is present in the warehouse when the new batch arrives, the deeper the first lane for the new batch will be.



They compared various methods to derive the lane depths and found that a limited number of lane depths provide a very close performance to the theoretical optimum. The optimal lane depths, selected from a limited number of depths, as computed by the **BLOCK** application, are shown in the next Figure.

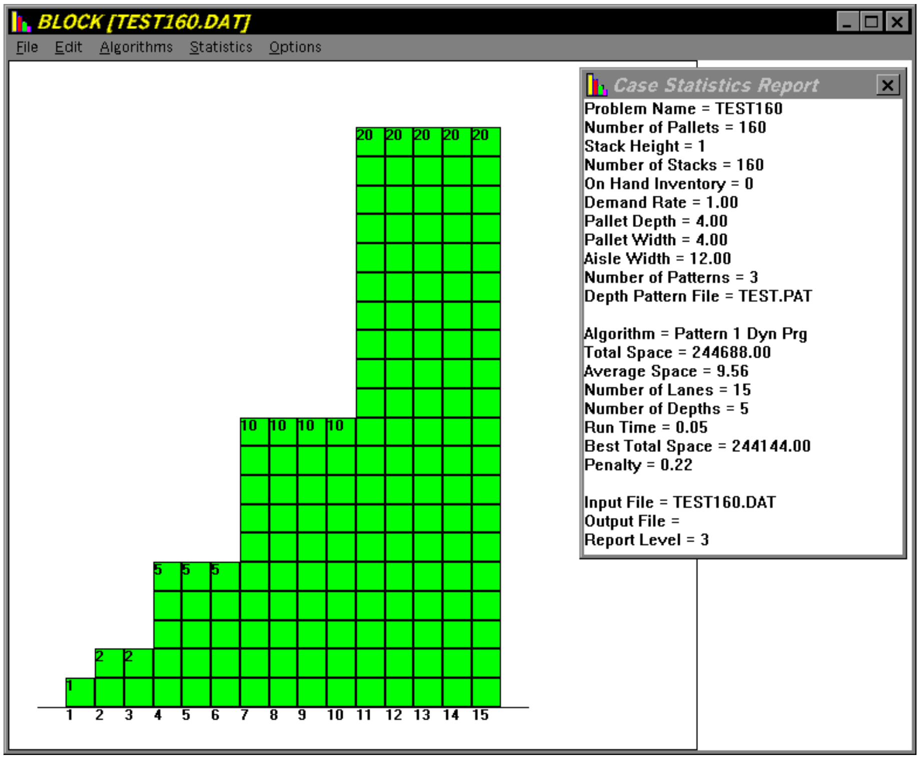


Figure 16.42. BLOCK Application Illustration

Depth Pattern Selection

Maximum 5 or 6 Different Depths

Range with Geometrical Series

Related to Order Batch Sizes

Maximum Depth ≈ $Q / 4z$

Experimental Comparison of Policies

Optimal (GR)

Triangle (TR)

Patterns (P2 & P5)

- P2 = (1, 2, 4, 8, 16, 32)
- P5 = (1, 2, 5, 10, 20, 40)

Discrete Equal (EQ)

Continuous Equal (CE)

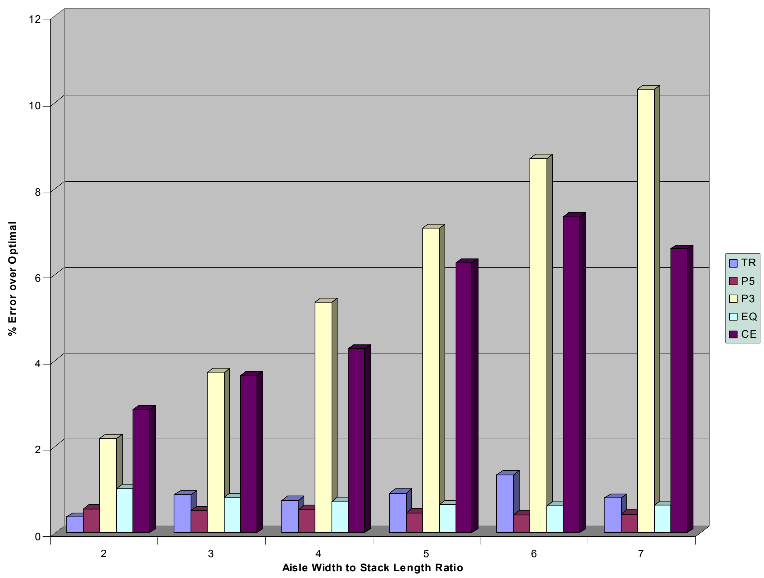


Figure 16.43. Influence of the Aisle to Pallet Ratio on Storage Policy Performance

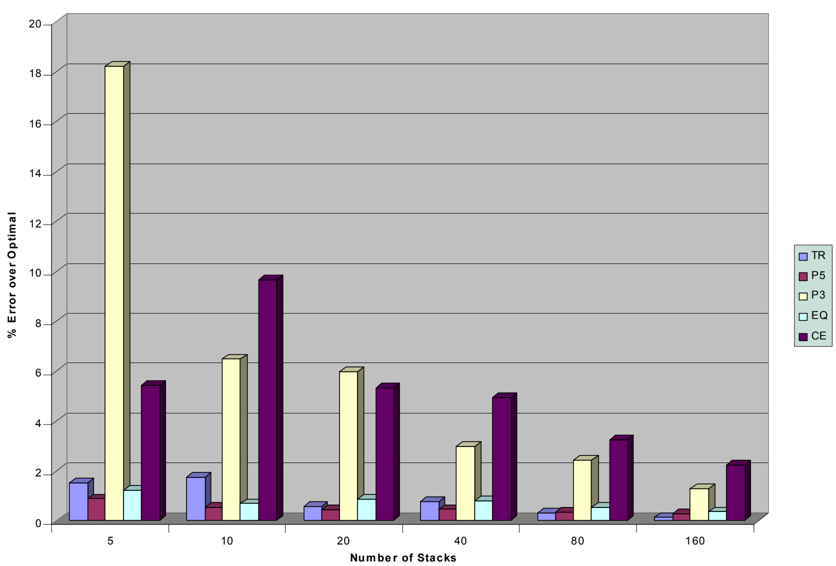


Figure 16.44. Effect of Batch Size on Storage Policy Performance

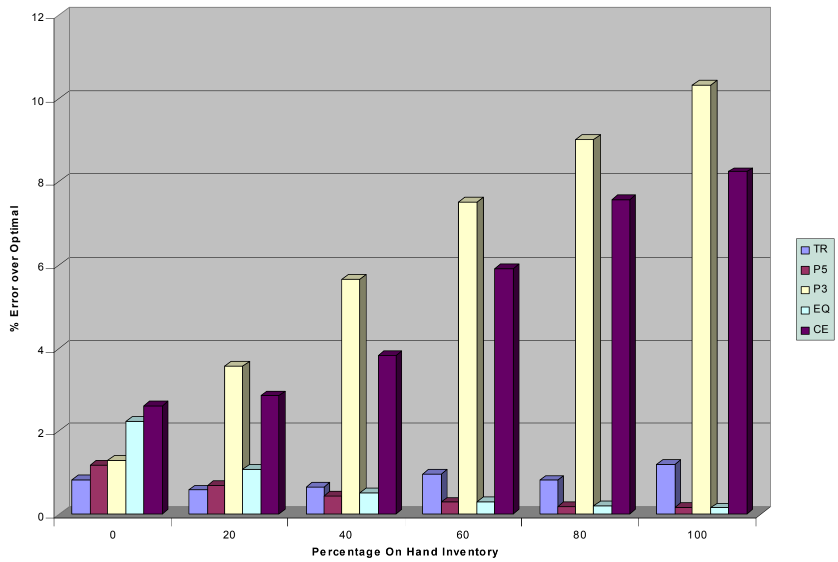


Figure 16.45. Effect on On Hand Inventory on Storage Policy Performance

Warehouse Layout Policy

1. Pick a geometrical series for depth pattern
2. Compute average number of required lanes for each product for each depth
3. Estimate the warehouse sharing factor
4. Compute required number of lanes for each depth
5. Successively round number of lanes to whole aisles for each product
6. Store the arriving batches based on pattern lane depths or in depth-proximity lanes

The successive rounding procedure is illustrated by the following example. The aisles all have 60 lanes. The lane depths considered are 4 and 8 stacks. The number of lanes required by all the products combined are 64 and 60 lanes for depths 4 and 8, respectively. Since all lanes on one side of the aisle must have the same depth to accommodate the travel of the vehicles, one side of the aisle will hold 64 lanes of 4 stacks deep. This is 4 lanes more than required. These 4 lanes of 4 stacks deep correspond to 2 lanes of 8 stacks deep. So the required number of lanes of 8 stacks deep is adjusted by subtracting these 2 lanes is now equal to 58 lanes. These 58 lanes require one side of the aisle. If more lane depths were available and required, the procedure would be repeated. So the overall layout for this example is a single aisle with 64 lanes of 4 deep on one side and 64 lanes of 8 deep on the other side.

Deep Lane Storage Systems

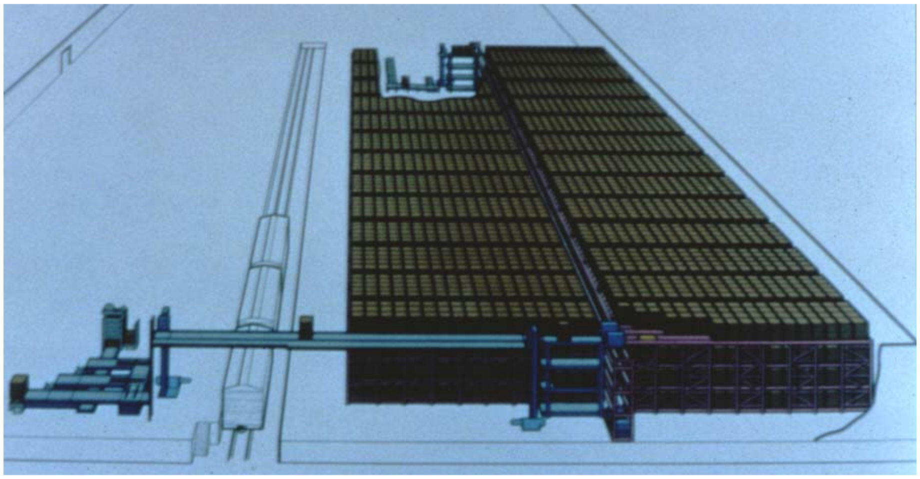


Figure 16.46. Deep Lane Storage System Illustration

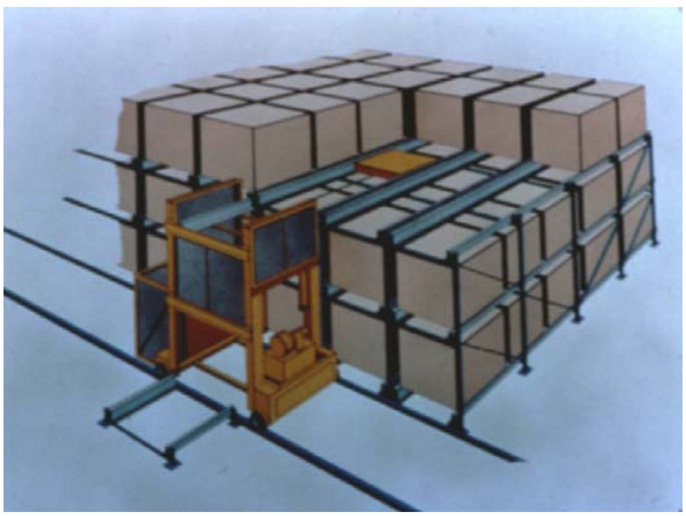


Figure 16.47. Deep Lane Storage Detail Illustration

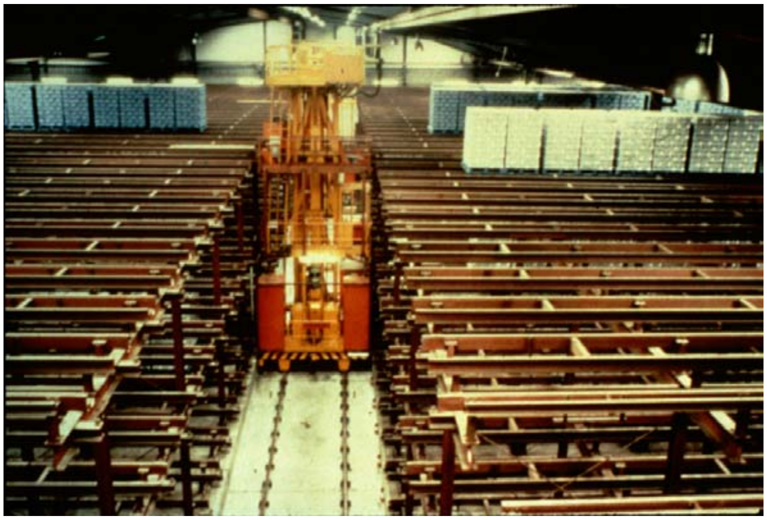


Figure 16.48. Deep Lane Storage System Example

Exercises

Comparison of Storage Policies Exercise 1

Four products are stored in the warehouse shown in Figure 16.49. Assume rectilinear distance between the dock centroid, indicated by the circle in the Figure 16.49, and the centroid of the storage bays. Furthermore, assume single command travel cycles. Each storage bay measures 20 by 20 feet. Eighty storage bays are available for storage. A product is replenished when its inventory reaches zero, i.e., there is no safety stock. The replenishment quantities, the number of demand operations per day, and the arrival day for each product are given in the Table 16.10. The warehouse is operating as a stationary cyclical process. The arrival day is the day in the cycle that the product gets replenished.

Table 16.10. Product Information

Product	Replenishment Quantity	Demand per Day	Arrival Day
A	12	4	1
B	28	7	1
C	24	4	3
D	16	4	2

Determine first if the travel independence condition is satisfied. Then determine the expected one way travel distance to each of the bays.

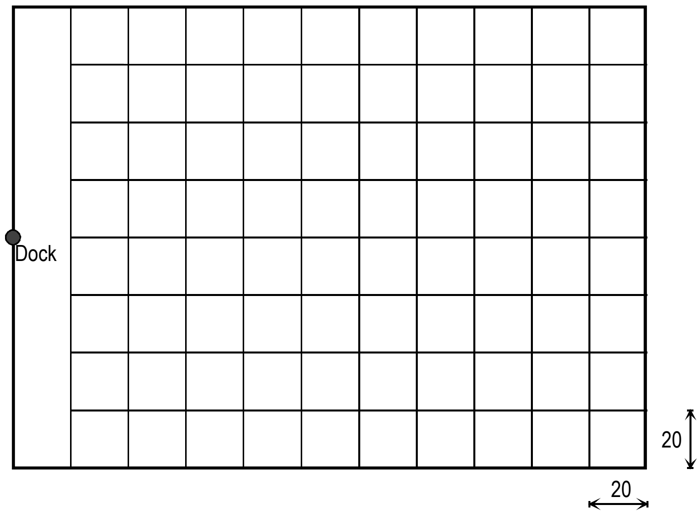


Figure 16.49. Warehouse Layout

Determine the optimal product dedicated storage layout that minimizes the expected travel distance per day. Compute the expected travel time per product and the total expected travel time for this warehouse layout. Assume that the products are stored by decreasing demand in the most desirable locations. Compute the expected travel time per product and the total expected travel time for this warehouse layout. Assume that the products are stored by increasing required storage in the most desirable locations. Compute the expected travel time per product and the total expected travel time for this warehouse layout. What are the required space penalty and travel time penalty (i.e. excess over the best policy) for each policy for this case. Summarize your answer in a clear table.

Assume that the arrival day for products B and C are swapped, i.e. product B now arrives on day 3 and product C now arrives on day 1 of the warehouse cycle. What are the required space penalty and travel time penalty (i.e. excess over the best policy) for each policy for this case. Summarize your answer in a clear table.

Suppose that random storage, rather than product dedicated storage, is used in this warehouse. Assume that replenishments for a day occur after all the demands for that day have been satisfied. What is the cycle for this warehouse operating under random storage policy? What is the number of units present in the warehouse at the end of each day of the cycle? What is the maximum number of storage bays required for storing the products using random storage? Show the warehouse layout for random storage at the end of day five of the cycle. Compute the expected travel time per day for every day in the cycle and the total expected travel time for this warehouse layout. What is the warehouse size, the sharing factor, and the balance of this warehouse system for the random storage policy? Discuss the advantages and disadvantages of random storage versus product dedicated storage.

This problem has been adapted from Tompkins and White (1984). The total expected travel time for product dedicated storage is 11,200. The required warehouse size for random storage is 69 and the total expected travel time is 8,897.

Storage Policy Comparison Exercise 2

Three products are stored in the warehouse shown in Figure 16.49. Products arrive at the receiving dock and depart through the shipping dock. The dock locations are indicated by the black circles in Figure 16.49. Assume rectilinear travel distance between the docks and the centroid of the storage bays. All material handling operations are executed with single command material handling cycles. A total of 48 storage bays are available and each storage bay measures 20 by 20 feet. The number of bays required for storage, the number of operations per day, and the arrival day for each product are given in the Table 16.11. The warehouse is operating as a stationary, cyclical process. The arrival day indicates the day in the warehouse cycle that the product gets replenished.

Table 16.11. Product Information

Product	Storage Bays	Demand per Day	Arrival Day
Variable	q	d	r
A	10	5	1
B	8	2	3
C	30	10	2

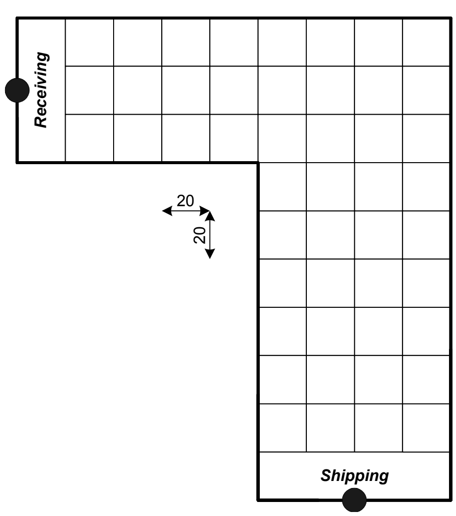


Figure 16.50. Warehouse Layout Distances

Determine the optimal product dedicated storage layout that minimizes the expected travel distance per day. Compute the expected travel time per product and the total expected travel time for this warehouse layout. Assume that the products are stored by decreasing demand in the most desirable locations. Compute the expected travel time per product and the total expected travel time for this warehouse layout. Assume that the products are stored by increasing required storage in the most desirable locations. Compute the expected travel time per product and the total expected travel time for this warehouse layout. What are the required space penalty and travel time penalty (i.e. excess over the best policy) for each policy for this case. Summarize your answer in a clear table.

Assume that the arrival day for products B and C are swapped, i.e. product B now arrives on day 3 and product C now arrives on day 1 of the warehouse cycle. What are the required space penalty and travel time penalty (i.e. excess over the best policy) for each policy for this case. Summarize your answer in a clear table. Discuss any differences or similarities between the previous two tables and explain the reason for the similarities and differences.

Storage Policy Comparison Exercise 3

You have been asked to redesign a unit load storage warehouse. Ten products are stored in the warehouse. The historic weekly demands for the ten products are given in Table 16.12. You can assume that the weekly demand for each product is normally distributed. Each product is replenished once a week. The marketing manager insists that enough inventories be held in the warehouse to cover 98 % of the weekly demand for each product.

Table 16.12. Weekly Product Demand

	Week												
Prod	1	2	3	4	5	6	7	8	9	10	11	12	13
A	36	45	37	33	35	43	39	42	41	42	35	37	45
B	5	5	9	6	7	8	12	8	10	5	10	10	10
C	14	11	15	6	12	14	4	22	5	11	12	13	13
D	33	43	40	37	45	34	37	43	42	31	36	32	28
E	21	17	23	16	26	18	20	16	10	13	12	14	17
F	72	78	75	77	73	83	83	78	88	82	75	87	76
G	8	5	5	3	10	10	8	6	3	4	8	9	5
H	11	12	15	19	16	14	12	15	13	9	17	16	8
I	56	50	45	49	53	35	45	58	37	57	60	55	55
J	20	19	23	19	27	29	31	14	13	18	19	25	17

The products are stored in a rack structure. Each unit load is stored on a 1200 by 1200 mm (millimeters) pallet. The effective width along the aisle of a storage location is 1400 mm, measured between the centroids of adjacent storage locations. The total depth of a storage location on one side of the aisle is 1300 mm. The effective height of a storage location is 1200 mm measured between the centroids of two storage locations that are above each other. The lowest level of storage locations can be assumed to be at ground level, i.e., no raising of the forks of the forklift trucks is required. There are storage locations on both sides of the aisles. There are three double side aisles. The rack has three storage levels or rows. The warehouse layout is illustrated in Figure 16.51, but your warehouse can have more or fewer storage locations in the aisles. Determining the number of columns to be used is part of your design task, but each aisle must have the same depth or number of columns.

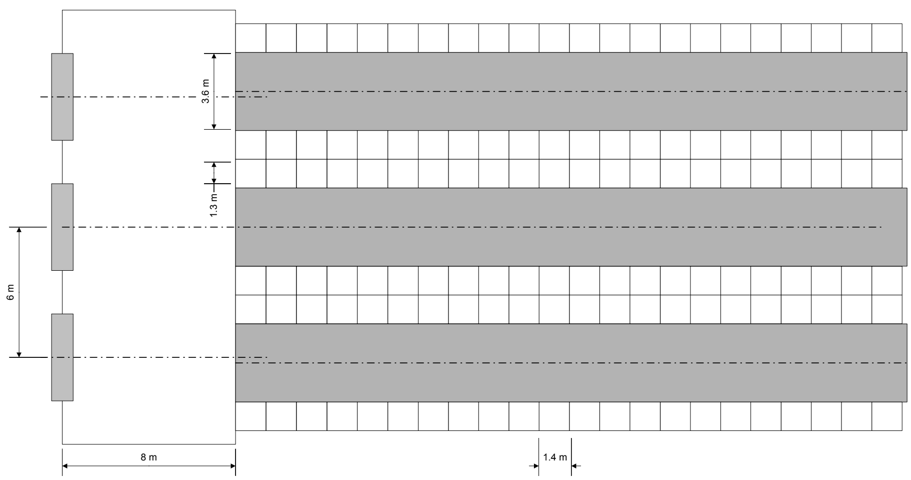


Figure 16.51. Storage Comparison Warehouse Layout 3

Receiving and shipping occurs through three truck doors at one side of the warehouse. The distance between truck doors is 6 m (meters), measured between the centroids of adjacent doors. Right behind the truck doors there is a 8 m wide combined shipping and receiving area. Behind this combined shipping and receiving area are three aisles. The aisles are oriented perpendicular to the shipping and receiving area. The centerline of the middle aisle is aligned with the centroid of the central truck door. The width of the travel aisle is 3.6 m. Storage and retrieval operations are executed by counterbalanced forklift trucks that operate in single command mode. It is assumed that products arrive and are shipped randomly from any of the truck doors. The horizontal speed of the forklifts is 2.75 m/s (meters per second) and the vertical lifting speed is 0.4 m/s. Due to saftely regulations, the forklift truck must have its forks in the lower position while traveling and cannot raise or lower its fork while traveling. The time to pick up or deposit a unit load in the rack is 30 seconds, respectively, and the average time to store or retrieve a unit load from a truck trailer is 60 seconds. It is assumed that the travels of the forklifts in the shipping and receiving area follow rectilinear paths. You can also assume that, when traveling in the storage aisles, the forklifts drive on the centerline of the aisle. To pickup or deposit a load in the rack, the forklifts must then drive to the side of the aisle and you must account for this travel time.

Your task is to design the warehouse and the rack system and specify the policy for the warehouse operations. This includes:

1. determining the number of columns or depth of the aisles,
2. determining the amount of inventories to be held for each product,
3. determining a storage policy for the warehouse operations, and
4. evaluating this storage policy with respect to the hours operated by the of forklift trucks in one week.

Your team should present no more than three designs. The storage policy is an integral part of a design, i.e., an otherwise identical physical aisle configuration with different storage policies counts as different designs. The warehouse manager likes the simplicity of a storage policy where storage locations are permanently assigned to a product and requests that at least one of the possible designs must be based on such a product dedicated storage policy. The warehouse manager also likes to evaluate a policy that leaves the storage decision to the forklift truck drivers and requests that at least one of the possible designs must be based on the closest-open-location (COL) storage policy. The company will then choose from the designs presented by your team based on their evaluation of the tradeoffs between storage location cost and operating costs (you do not have to make this tradeoff).

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